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2×6×*n*阶级联蛛网电阻网络电性质研究

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摘要:蛛网模型因具有良好的拓扑特性,广泛应用于材料设计和模拟计算等领域。本文研究了一类以蛛网模型为基本单元组成的2×6×*n*阶级联蛛网电阻网络的电特性。由于这类网络结构复杂,用传统方法求解较为困难,因此,本文采用一种优化的递推-变换方法。首先,将级联网络等效变换为单级网络;其次,建立单级网络非线性递推方程;然后,将非线性递推方程转变为线性递推方程;最后,借助线性递推方程求解待解电阻网络的等效电阻。本方法避免了传统方法中复杂的矩阵运算,具有简单便捷的特点,还可推广应用到其他二端级联网络。

关键词:2×6×*n* 阶电路;级联蛛网;电阻网络;递推-变换方法
中图分类号:O441.1;TN711.3　　**文献标志码**:A

Study on the electrical properties of a 2×6×*n* cascaded cobweb resistor network

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Abstract: Due to their excellent topological properties, cobweb models are widely used in fields such as material design and simulation calculations. This study focuses on the electrical characteristics of a 2×6×*n* cascaded cobweb resistor network constructed with cobweb models as basic units. The complex structure of such networks makes it difficult to solve using traditional methods. To address this issue, an optimized recurrence-transformation method is proposed in this paper. First, the cascaded network is equivalently transformed into a single-stage network. Second, a nonlinear recurrence equation for the single-stage network is established. Third, the nonlinear recurrence equation is converted into a linear recurrence equation. Finally, the equivalent resistance of the target resistor network is solved with the help of the linear recurrence equation. This method avoids complex matrix operations involved in traditional methods and features simplicity and convenience. Additionally, it can be extended to other two-terminal cascaded networks.

Keywords: 2×6×*n* order circuit; cascaded cobweb; resistor network; recursion-transform method
CLC: O441.1, TN711.3　　**DC**: A

0　引　言

在物理学、材料科学、图论学等领域,一些用传统方法无法解决的复杂理论或技术问题,借助构建电阻网络模型进行模拟的方法得以解决。例如:Lin等[1]利用电阻网络成功模拟了凝聚态物理学中的各

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种拓扑态;Lin等[2]运用扩展的电阻网络模型解释了二维海森堡自旋晶格的磁电阻效应。

电阻网络研究的理论基础是基尔霍夫电流定律和电压定律。人们在求解基尔霍夫电流和电压方程时,引入各种数学计算技巧,衍生出各种电阻网络计算方法,如格林函数计算方法[3]、拉普拉斯矩阵变换方法[4]和递推-变换方法[5-9]。

蛛网模型是一种仿照自然界蜘蛛网构建的网络模型,融合了星形网络、树形网络、环形网络和网格网络的拓扑特点,广泛应用于材料设计等领域。近年来,随着纳米制备技术的发展,许多二维纳米蛛网材料被成功制备出,并且在电子材料、环境防护、生物工程等领域展现出广阔的应用前景。例如,张世超[10]利用新型静电喷网技术制备出一种具有蛛网结构的二维网状超细纳米纤维材料,有望成为一种理想的空气过滤材料。

由于蛛网模型在解决实际问题中的出色表现以及潜在的应用价值,以蛛网模型为基本结构组成的各种蛛网电阻网络引起了人们极大的兴趣。谭志中团队[11-14]对2×n阶、3×n阶、4×n阶以及m×n阶蛛网电阻网络的等效电阻进行了研究;Izmailian团队[15]对蛛网电阻网络中心节点和边界节点间的等效电阻问题进行了探究。

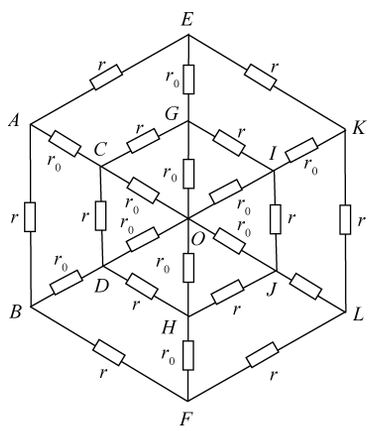
鉴于大多数蛛网材料都具有六边形蛛网结构,为研究这类材料的电性质,本文建立了六边形蛛网结构的电阻网络1×6×n和2×6×n阶级联蛛网电阻网络模型。前者是具有单六边形结构的蛛网电阻网络,其电学特性的研究已由本课题组完成[16],后者是具有双六边形结构的蛛网电阻网络,是对前期工作的拓展与延伸。本文将在先前工作的基础上,进一步研究具有双六边形蛛网结构的2×6×n阶级联蛛网电阻网络的电特性。

1　计算方法

递推-变换方法是一种求解多种类型周期性电阻网络等效电阻的有效方法[17-21]。该方法主要包括建立任意相邻子网络的递推关系、将递推关系线性化、求解线性递推关系的通解以及利用边界条件求需求端的等效电阻等步骤。理论上,该方法可以求解任意周期性电阻网络的等效电阻,包括本文计算的级联蛛网电阻网络。应用该方法时,面临如何确定线性变换矩阵以及将非线性递推关系线性化等繁杂的矩阵变换问题。为避开这些问题,本文另辟蹊径,采用先简化电路,后建立递推关系,然后将递推关系线性化后再求解的方法,称这一方法为优化的递推-变换方法。运用该方法计算本文的级联蛛网电阻网络等效电阻的具体步骤如下:首先,将所求解的级联电阻网络等效变换为一单级电阻网络;其次,根据基尔霍夫电路定律构建单级电阻网络的非线性递推关系;再次,将非线性递推关系转变为线性递推关系;最后,通过求解线性递推关系获得所求解电阻网络的等效电阻。下面将运用该方法计算2×6×n阶级联蛛网电阻网络的等效电阻。

2　2×6×n阶级联蛛网电阻网络的等效电阻

根据文献[11]的定义,图1的电路称为2×6阶蛛网电阻网络。图2定义2×6×n阶级联蛛网电阻网络,其含有2个不同的电阻元素r_0、r及1个负载电阻R_0。下面将运用上述优化的递推-变换方法研究节点A_n与B_n、C_n与D_n、E_n与F_n及G_n与H_n间的等效电阻。首先求解图2中节点A_n与B_n间的等效电阻。



注:r_0为各经线电阻阻值;r为各纬线电阻阻值;
A~L、O为电路节点。

图1　2×6阶蛛网电阻网络

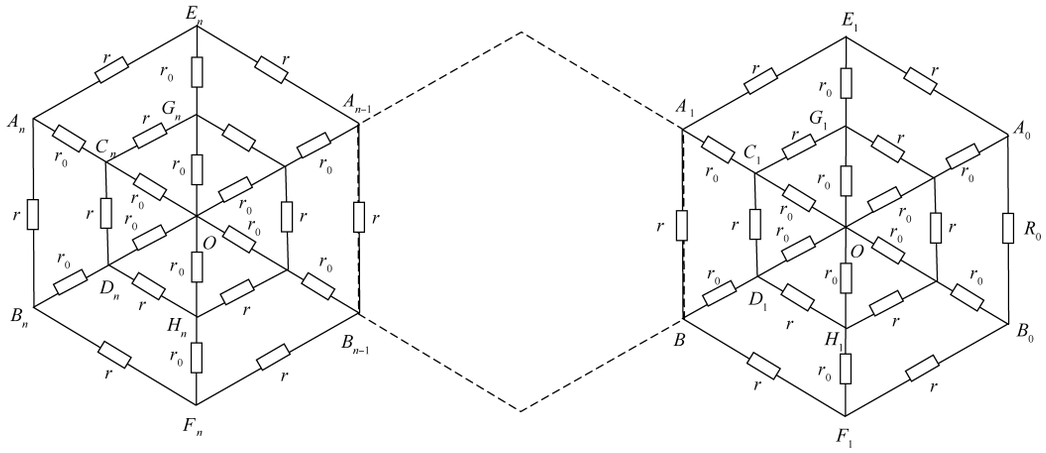
Figure 1　2×6 cobweb resistor network

2.1　将级联网络等效变换为单级网络

图2所示的n阶电路是由第1至第n−1阶构成的电路和第n阶电路2部分组成。如果第1至第n−1阶构成的电路中节点A_{n-1}与B_{n-1}间的等效电阻为R_{n-1}^{AB},则整个电路可看作是由等效电阻R_{n-1}^{AB}和第n阶电路组成,所以图2的电路可以等效为图3的电路。

2.2　构建非线性递推关系

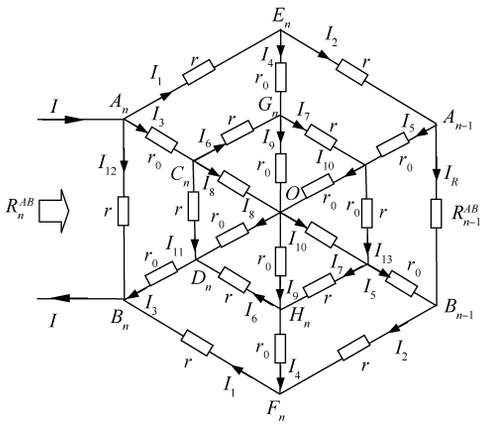
在图3所示的等效电路中,为求解节点A_n与B_n间的等效电阻,假设恒定电流(I)从节点A_n输入,再从节点B_n输出,流经各个支路的电流如图3所示。对各独立回路应用基尔霍夫电压定律得:



注:A_i~H_i为第i阶电路节点($i=0,1,\cdots,n$)。

图2　2×6×n 阶级联蛛网电阻网络

Figure 2　2×6×n cascaded cobweb resistor network



注:R_n^{AB}和R_{n-1}^{AB}分别表示A_n与B_n、A_{n-1}与B_{n-1}间的等效电阻;I表示总电流;I_1~I_{13}和I_R为流经各支路的电流;A_n~H_n为第n阶电路节点;A_{n-1}和B_{n-1}为第$n-1$阶电路节点。

图3　2×6×n 阶级联蛛网电阻网络的等效网络

Figure 3　equivalent network of 2×6×n cascaded cobweb resistor network

$$2r_0I_3+rI_{11}-rI_{12}=0, \tag{1}$$

$$rI_1-r_0I_3+r_0I_4-rI_6=0, \tag{2}$$

$$rI_2-r_0I_4+r_0I_5-rI_7=0, \tag{3}$$

$$2r_0I_5+rI_{13}-R_{n-1}^{AB}I_R=0, \tag{4}$$

$$2r_0I_8-rI_{11}=0, \tag{5}$$

$$rI_6-r_0I_8+r_0I_9=0, \tag{6}$$

$$rI_7-r_0I_9+r_0I_{10}=0, \tag{7}$$

$$2r_0I_{10}-rI_{13}=0。 \tag{8}$$

对各独立节点应用基尔霍夫电流定律得:

$$I-I_1-I_3-I_{12}=0, \tag{9}$$

$$I_1-I_2-I_4=0, \tag{10}$$

$$I_2-I_5-I_R=0, \tag{11}$$

$$I_3-I_6-I_8-I_{11}=0, \tag{12}$$

$$I_4+I_6-I_7-I_9=0, \tag{13}$$

$$I_5+I_7-I_{10}-I_{13}=0。 \tag{14}$$

联立方程(1~14)求解出

$$I_{12}=\left(\frac{\alpha_1R_{n-1}^{AB}+\mu_1}{\beta_1R_{n-1}^{AB}+\nu_1}\right)I, \tag{15}$$

式中:

$$\alpha_1=24r_0^6+148r_0^5r+266r_0^4r^2+184r_0^3r^3+50r_0^2r^4+4r_0r^5, \tag{16}$$

$$\beta_1=24r_0^6+184r_0^5r+395r_0^4r^2+344r_0^3r^3+131r_0^2r^4+20r_0r^5+r^6, \tag{17}$$

$$\mu_1=96r_0^6r+352r_0^5r^2+372r_0^4r^3+144r_0^3r^4+16r_0^2r^5, \tag{18}$$

$$\nu_1=120r_0^6r+500r_0^5r^2+638r_0^4r^3+328r_0^3r^4+66r_0^2r^5+4r_0r^6。 \tag{19}$$

由图3可知,节点A_n与B_n间的等效电阻为

$$R_n^{AB}=\frac{U_n^{AB}}{I}=\frac{I_{12}r}{I}。 \tag{20}$$

将式(15)代入式(20)得

$$R_n^{AB}=\left(\frac{\alpha_1R_{n-1}^{AB}+\mu_1}{\beta_1R_{n-1}^{AB}+\nu_1}\right)r, \tag{21}$$

式(21)便是求解R_n^{AB}的非线性递推方程。

2.3　将非线性递推关系线性化

方程(21)是一个非线性递推方程,要求解此方程,可通过变量变换方法将其转变为线性方程。为此,引入一个数列$\{\xi_n\}$,使其满足以下变换关系

$$R_n^{AB}=d_1\frac{\xi_{n+1}}{\xi_n}-\frac{\nu_1}{\beta_1}, \tag{22}$$

式中$d_1=\frac{\sqrt{(\beta_1\mu_1-\alpha_1\nu_1)r}}{\beta_1}$。将式(22)代入式(21),

整理得

$$\xi_{n+1} = h\xi_n + \xi_{n-1}, \tag{23}$$

式中 $h = \dfrac{\alpha_1 r + v_1}{\sqrt{(\beta_1\mu_1 - \alpha_1 v_1)r}}$,方程(23)便是一个线性递推方程。

2.4 求线性递推关系的解析解及等效电阻 R_n^{AB}

采用特征根法解方程(23),其特征方程为

$$\chi^2 = h\chi + 1。 \tag{24}$$

解出方程(24)的根为:

$$\chi_1 = \frac{\sqrt{h^2+4}}{2} + \frac{h}{2}, \tag{25}$$

$$\chi_2 = -\frac{\sqrt{h^2+4}}{2} + \frac{h}{2}, \tag{26}$$

则有 $\chi_1 + \chi_2 = h$,$\chi_1\chi_2 = -1$,代入式(23)得

$$\xi_{n+1} = (\chi_1 + \chi_2)\,\xi_n - \chi_1\chi_2\xi_{n-1}。 \tag{27}$$

根据式(27)得:

$$\xi_{n+1} - \chi_1\xi_n = \chi_2(\xi_n - \chi_1\xi_{n-1}) = \chi_2^{\,2}(\xi_{n-1} - \chi_1\xi_{n-2}) = \cdots = \chi_2^{\,n}(\xi_1 - \chi_1\xi_0), \tag{28}$$

$$\xi_{n+1} - \chi_2\xi_n = \chi_1(\xi_n - \chi_2\xi_{n-1}) = \chi_1^{\,2}(\xi_{n-1} - \chi_2\xi_{n-2}) = \cdots = \chi_1^{\,n}(\xi_1 - \chi_2\xi_0)。 \tag{29}$$

将式(29)减去式(28)得

$$\xi_n = \frac{1}{\chi_1 - \chi_2}\left[(\xi_1 - \chi_2\xi_0)\chi_1^{\,n} - (\xi_1 - \chi_1\xi_0)\chi_2^{\,n}\right] = \xi_1\left(\frac{\chi_1^{\,n} - \chi_2^{\,n}}{\chi_1 - \chi_2}\right) + \xi_0\left(\frac{\chi_1^{\,n-1} - \chi_2^{\,n-1}}{\chi_1 - \chi_2}\right)。 \tag{30}$$

选定 ξ_n 的初始项为 $\xi_0 = d_1$,则根据式(22)得:$\xi_1 = R_0 + \dfrac{v_1}{\beta_1}$,并设 $c_1 = R_0 + \dfrac{v_1}{\beta_1}$。将 ξ_0、ξ_1 分别代入式(30)得

$$\xi_n = c_1\left(\frac{\chi_1^{\,n} - \chi_2^{\,n}}{\chi_1 - \chi_2}\right) + d_1\left(\frac{\chi_1^{\,n-1} - \chi_2^{\,n-1}}{\chi_1 - \chi_2}\right)。 \tag{31}$$

引入函数

$$F_n = \frac{\chi_1^{\,n} - \chi_2^{\,n}}{\chi_1 - \chi_2}, \tag{32}$$

则式(31)变为

$$\xi_n = c_1F_n + d_1F_{n-1}。 \tag{33}$$

将式(33)代入式(22)得

$$R_n^{AB} = \frac{a_1F_{n+1} + b_1F_n}{c_1F_n + d_1F_{n-1}} - \frac{v_1}{\beta_1}, \tag{34}$$

式中

$$a_1 = c_1d_1, b_1 = d_1^2, c_1 = R_0 + \frac{v_1}{\beta_1}, d_1 = \frac{\sqrt{(\beta_1\mu_1 - \alpha_1 v_1)r}}{\beta_1}。 \tag{35}$$

式(34)便是图2所示的2×6×n阶级联蛛网电阻网络节点 A_n 与 B_n 间等效电阻的解析式。

2.5 求等效电阻 R_n^{CD}、R_n^{EF} 和 R_n^{GH}

用同样的方法可求解出图2网络中节点 C_n 与 D_n、E_n 与 F_n 及 G_n 与 H_n 间的等效电阻 R_n^{CD}、R_n^{EF} 和 R_n^{GH},其表达式分别为

$$R_n^{CD} = \left(\frac{\alpha_2R_{n-1}^{AB} + \mu_2}{\beta_1R_{n-1}^{AB} + v_1}\right)r = \frac{a_2F_{n-1} + b_2F_{n-2}}{c_1F_n + d_1F_{n-1}} + \frac{v_2}{\beta_1}, \tag{36}$$

式中:

$$\alpha_2 = 20r_0^6 + 130r_0^5r + 206r_0^4r^2 + 122r_0^3r^3 + 28r_0^2r^4 + 2r_0r^5, \tag{37}$$

$$\mu_2 = 100r_0^6r + 300r_0^5r^2 + 264r_0^4r^3 + 84r_0^3r^4 + 8r_0^2r^5, \tag{38}$$

$$a_2 = \frac{c_1d_2^2}{d_1}, b_2 = d_2^2, d_2 = \frac{\sqrt{(\beta_1\mu_2 - \alpha_2 v_1)r}}{\beta_1}, v_2 = \alpha_2 r; \tag{39}$$

$$R_n^{EF} = \left(\frac{\alpha_3R_{n-1}^{AB} + \mu_3}{\beta_1R_{n-1}^{AB} + v_1}\right)r_0 = \frac{a_3F_{n-1} + b_3F_{n-2}}{c_1F_n + d_1F_{n-1}} + \frac{v_3}{\beta_1}, \tag{40}$$

式中:

$$\alpha_3 = 72r_0^5r + 324r_0^4r^2 + 480r_0^3r^3 + 280r_0^2r^4 + 60r_0r^5 + 4r^6, \tag{41}$$

$$\mu_3 = 144r_0^5r^2 + 504r_0^4r^3 + 516r_0^3r^4 + 184r_0^2r^5 + 16r_0r^6, \tag{42}$$

$$a_3 = \frac{c_1d_3^2}{d_1}, b_3 = d_3^2, d_3 = \frac{\sqrt{(\beta_1\mu_3 - \alpha_3 v_1)r_0}}{\beta_1}, v_3 = \alpha_3 r_0; \tag{43}$$

$$R_n^{GH} = \left(\frac{\alpha_4R_{n-1}^{AB} + \mu_4}{\beta_1R_{n-1}^{AB} + v_1}\right)r_0 = \frac{a_4F_{n-1} + b_4F_{n-2}}{c_1F_n + d_1F_{n-1}} + \frac{v_4}{\beta_1}, \tag{44}$$

式中:

$$\alpha_4 = 36r_0^5r + 210r_0^4r^2 + 304r_0^3r^3 + 162r_0^2r^4 + 32r_0r^5 + 2r^6, \tag{45}$$

$$\mu_4 = 180r_0^5r^2 + 420r_0^4r^3 + 332r_0^3r^4 + 100r_0^2r^5 + 8r_0r^6, \tag{46}$$

$$a_4 = \frac{c_1d_4^2}{d_1}, b_4 = d_4^2, d_4 = \frac{\sqrt{(\beta_1\mu_4 - \alpha_4 v_1)r_0}}{\beta_1}, v_4 = \alpha_4 r_0。 \tag{47}$$

此外,c_1、d_1 如式(35)所示,β_1、v_1 如式(17)和(19)所示。

3 特定情形讨论

对图2所示的2×6×n阶级联蛛网电阻网络,根据式(34、36、40、44),讨论3种特定情形。

3.1 n=0

当 $n = 0$ 时,图2的电路变为只有1个负载电阻

R_0组成的电路,相当于节点A_n与B_n间只连接R_0。由式(32)可得

$$F_0 = 0, F_1 = 1, F_{-1} = 1。\tag{48}$$

将式(48)、(35)代入式(34)得

$$R_0^{AB} = \frac{a_1F_1 + b_1F_0}{c_1F_0 + d_1F_{-1}} - \frac{v_1}{\beta_1} = R_0。\tag{49}$$

这与实际情形一致。此情形下不存在R_n^{CD}、R_n^{EF}和R_n^{GH}。

3.2 n=1

当n = 1,且R_0 = r时,图2所示的电阻网络转变为图1所示的2×6阶蛛网电阻网络,由式(32)得

$$F_0 = 0, F_1 = 1, F_2 = \frac{\alpha_1 r + v_1}{\sqrt{(\beta_1\mu_1 - \alpha_1 v_1)r}}。\tag{50}$$

将式(50)、(35)代入式(34)得

$$R_1^{AB} = \frac{a_1F_2 + b_1F_1}{c_1F_1 + d_1F_0} - \frac{v_1}{\beta_1} = \frac{(\beta_1\mu_1 - \alpha_1 v_1)r}{\beta_1(\beta_1 r + v_1)} + \frac{\alpha_1 r}{\beta_1}。\tag{51}$$

用类似的方法可得:

$$R_1^{CD} = \frac{a_2F_0 + b_2F_{-1}}{c_1F_1 + d_1F_0} + \frac{v_2}{\beta_1} = \frac{(\beta_1\mu_2 - \alpha_2 v_1)r}{\beta_1(\beta_1 r + v_1)} + \frac{\alpha_2 r}{\beta_1},\tag{52}$$

$$R_1^{EF} = \frac{a_3F_0 + b_3F_{-1}}{c_1F_1 + d_1F_0} + \frac{v_3}{\beta_1} = \frac{(\beta_1\mu_3 - \alpha_3 v_1)r_0}{\beta_1(\beta_1 r + v_1)} + \frac{\alpha_3 r_0}{\beta_1},\tag{53}$$

$$R_1^{GH} = \frac{a_4F_0 + b_4F_{-1}}{c_1F_1 + d_1F_0} + \frac{v_4}{\beta_1} = \frac{(\beta_1\mu_4 - \alpha_4 v_1)r_0}{\beta_1(\beta_1 r + v_1)} + \frac{\alpha_4 r_0}{\beta_1}。\tag{54}$$

式(51~54)便是图1所示的2×6阶蛛网电阻网络中2节点间等效电阻的解析式。

3.3 $n \to \infty$

由式(25)和(26)可知$\chi_2 = -\frac{1}{\chi_1}$,代入式(32)得

$$F_n = \frac{\chi_1^n - (-1)^n\chi_1^{-n}}{\chi_1 + \chi_1^{-1}}。\tag{55}$$

再将式(55)代入式(34)得

$$R_n^{AB} = \frac{a_1\left(1 - (-1)^{n+1}\frac{1}{\chi_1^{2n+2}}\right) + b_1\left(\frac{1}{\chi_1} - (-1)^n\frac{1}{\chi_1^{2n+1}}\right)}{c_1\left(\frac{1}{\chi_1} - (-1)^n\frac{1}{\chi_1^{2n+1}}\right) + d_1\left(\frac{1}{\chi_1^2} - (-1)^{n-1}\frac{1}{\chi_1^{2n}}\right)} - \frac{v_1}{\beta_1}。\tag{56}$$

由式(56)可得

$$R_\infty^{AB} = \lim_{n\to\infty} R_n^{AB} = \frac{a_1 + b_1\chi_1^{-1}}{c_1\chi_1^{-1} + d_1\chi_1^{-2}} - \frac{v_1}{\beta_1}。\tag{57}$$

同理可推导出:

$$R_\infty^{CD} = \lim_{n\to\infty} R_n^{CD} = \frac{a_2 + b_2\chi_1^{-1}}{c_1\chi_1 + d_1} + \frac{v_2}{\beta_1},\tag{58}$$

$$R_\infty^{EF} = \lim_{n\to\infty} R_n^{EF} = \frac{a_3 + b_3\chi_1^{-1}}{c_1\chi_1 + d_1} + \frac{v_3}{\beta_1},\tag{59}$$

$$R_\infty^{GH} = \lim_{n\to\infty} R_n^{EF} = \frac{a_4 + b_4\chi_1^{-1}}{c_1\chi_1 + d_1} + \frac{v_4}{\beta_1}。\tag{60}$$

式(57~60)分别是图2所示的2×6×n阶级联蛛网电阻网络节点A_n与B_n、C_n与D_n、E_n与F_n及G_n与H_n间的等效电阻当$n \to \infty$时的解析式。当$n \to \infty$,此网络各节点间的等效电阻都有确定的极限值。

4 2×6×n阶级联蛛网电阻网络等效电阻特性讨论

2×6×n阶级联蛛网电阻网络的等效电阻由电路参数r_0、r及电路阶数n决定。下面讨论等效电阻随n的变化特性。为方便起见,取r_0= r,R_0= r。

4.1 R_n^{AB}随n的变化特性

把r_0=r分别代入式(16~19)得:

$$\alpha_1 = 636r^6, \mu_1 = 980r^7,\tag{61}$$

$$\beta_1 = 1\,099r^6, v_1 = 1\,656r^7。\tag{62}$$

把式(61)和(62)代入$h = \frac{\alpha_1 r + v_1}{\sqrt{(\beta_1\mu_1 - \alpha_1 v_1)r}}$,计算出$h = \frac{1\,146}{\sqrt{5\,951}}$,再分别代入式(25)和(26)得

$$\chi_1 = \frac{1\,146 + 4\sqrt{83\,570}}{\sqrt{23\,804}}, \chi_2 = \frac{1\,146 - 4\sqrt{83\,570}}{\sqrt{23\,804}}。\tag{63}$$

把式(61)和(62)代入式(35)分别计算得到

$$a_1 = \frac{5\,510\sqrt{5\,951}}{1\,207\,801}r^2, b_1 = \frac{47\,608}{2\,415\,602}r^2,$$

$$c_1 = \frac{5\,510}{2\,198}r, d_1 = \frac{2\sqrt{5\,951}}{1\,099}r。\tag{64}$$

将式(62)、(64)和(32)代入式(34)得

$$\frac{R_n^{AB}}{r} = \frac{e_1(\chi_1^{n+1} - \chi_2^{n+1}) + f_1(\chi_1^n - \chi_2^n)}{e_0(\chi_1^n - \chi_2^n) + f_0(\chi_1^{n-1} - \chi_2^{n-1})} - g^{AB},\tag{65}$$

式中:e_0 = 3 327 491 755$\sqrt{5\,951}$;f_0 = 14 375 247 502;e_1 = 36 036 220 990;f_1 = 26 160 596$\sqrt{5\,951}$;$g^{AB} = \frac{1\,656}{1\,099}$。式(65)是图2所示的2×6×$n$阶级联蛛网电阻网络节点$A_n$与$B_n$间等效电阻随$n$变化的解析式。

根据式(65),可作出R_n^{AB}/r随n变化的曲线,如图4所示。当n=1时,R_1^{AB}/r = 0.586 569 872 958 258,这与式(51)的结果一致。把式(62)、(63)和(64)代

入式(57)可得$R_{\infty}^{AB}/r \approx 0.588\,115\,605\,664\,112$。当$n$=7时,$R_n^{AB}/r$已经接近$R_{\infty}^{AB}/r$。

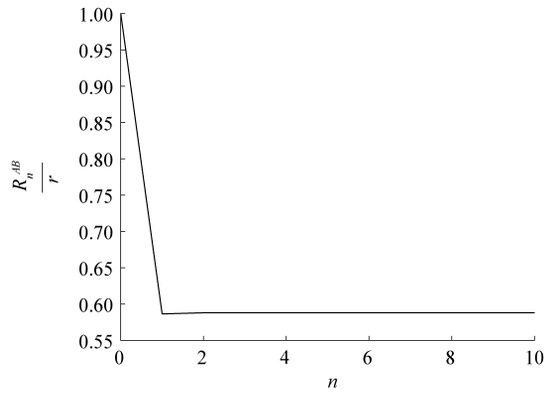


图4　R_n^{AB}/r随n变化的曲线

Figure 4　R_n^{AB}/r versus n

4.2　R_n^{CD}、R_n^{EF}和R_n^{GH}随n的变化特性

运用与推导R_n^{AB}/r相同的做法,可推导出图2所示的2×6×n阶级联蛛网电阻网络节点C_n与D_n、E_n与F_n及G_n与H_n间的等效电阻R_n^{CD}/r、R_n^{EF}/r和R_n^{GH}/r随n变化的解析式分别为:

$$\frac{R_n^{CD}}{r}=\frac{e_2\left(\chi_1^{n-1}-\chi_2^{n-1}\right)+f_2\left(\chi_1^{n-2}-\chi_2^{n-2}\right)}{e_0\left(\chi_1^{n}-\chi_2^{n}\right)+f_0\left(\chi_1^{n-1}-\chi_2^{n-1}\right)}+g^{CD},\quad(66)$$

式中:$e_0=3\,327\,491\,755\sqrt{5\,951}$;$f_0=14\,375\,247\,502$;$e_2=-15\,750\,329\,490$;$f_2=-11\,433\,996\sqrt{5\,951}$;$g^{CD}=\frac{508}{1\,099}$。

$$\frac{R_n^{EF}}{r}=\frac{e_3\left(\chi_1^{n-1}-\chi_2^{n-1}\right)+f_3\left(\chi_1^{n-2}-\chi_2^{n-2}\right)}{e_0\left(\chi_1^{n}-\chi_2^{n}\right)+f_0\left(\chi_1^{n-1}-\chi_2^{n-1}\right)}+g^{EF},\quad(67)$$

式中:$e_0=3\,327\,491\,755\sqrt{5\,951}$;$f_0=14\,375\,247\,502$;$e_3=-789\,157\,512\,290$;　$f_3=-572\,891\,116\sqrt{5\,951}$;$g^{EF}=\frac{1\,220}{1\,099}$。

$$\frac{R_n^{GH}}{r}=\frac{e_4\left(\chi_1^{n-1}-\chi_2^{n-1}\right)+f_4\left(\chi_1^{n-2}-\chi_2^{n-2}\right)}{e_0\left(\chi_1^{n}-\chi_2^{n}\right)+f_0\left(\chi_1^{n-1}-\chi_2^{n-1}\right)}+g^{GH},\quad(68)$$

式中:$e_0=3\,327\,491\,755\sqrt{5\,951}$;$f_0=14\,375\,247\,502$;$e_4=-139\,906\,040\,960$;　$f_4=-101\,565\,184\sqrt{5\,951}$;$g^{GH}=\frac{746}{1\,099}$。

根据式(66~68),可分别作出R_n^{CD}/r、R_n^{EF}/r及R_n^{GH}/r随n变化的曲线,如图5所示。当n=1时,$R_1^{CD}/r=0.458\,802\,177\,858\,439$,$R_1^{EF}/r=0.937\,931\,034\,482\,757$,$R_1^{GH}/r=0.648\,275\,862\,068\,965$,这分别与式(52~54)结果一致。把相关数据代入式(58~60)可分别计算出$R_{\infty}^{CD}/r\approx 0.458\,126\,585\,391\,975$、$R_{\infty}^{EF}/r\approx 0.904\,081\,020\,710\,348$、$R_{\infty}^{GH}/r\approx 0.642\,274\,751\,594\,078$。当$n$=7或8时,$R_n^{CD}/r$、$R_n^{EF}/r$和$R_n^{GH}/r$分别接近$R_{\infty}^{CD}/r$、$R_{\infty}^{EF}/r$和$R_{\infty}^{GH}/r$。

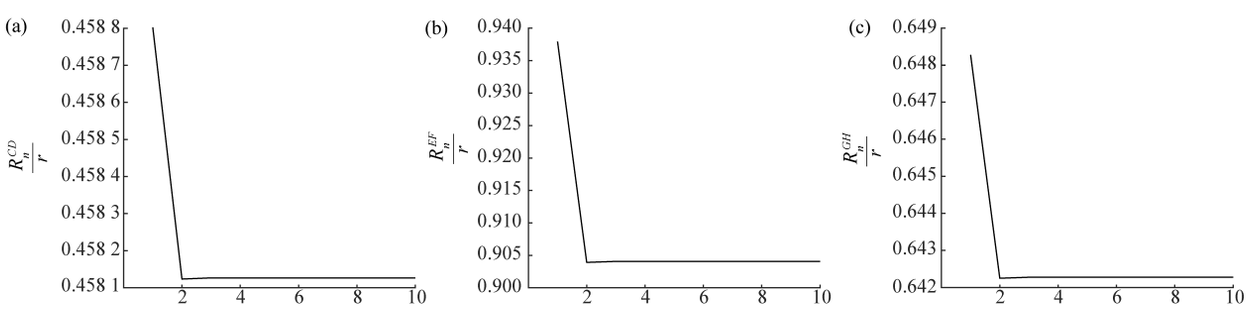


图5　各节点间等效电阻随n变化的曲线

(a)R_n^{CD}/r随n变化;(b)R_n^{EF}/r随n变化;(c)R_n^{GH}/r随n变化

Figure 5　equivalent resistance between nodes varying with n

(a)R_n^{CD}/r versus n;(b)R_n^{EF}/r versus n;(c)R_n^{GH}/r versus n

5　结　论

本文采用优化的递推-变换方法研究了2×6×n阶级联蛛网电阻网络的电特性。通过建立等效电路、构建等效电路的非线性递推关系、将非线性递推关系线性化、求解线性递推方程等步骤,得到了所求解电路网络的等效电阻。相较于前期的递推-变换方法,该方法采取了先简化电路后求解的方法,使得计算过程大为简化。计算结果表明,这类电阻网络2节点间的等效电阻随阶数n的变化而变化,并且当$n\to\infty$,分别收敛于R_{∞}^{AB}/r、R_{∞}^{CD}/r、R_{∞}^{EF}/r和R_{∞}^{GH}/r。计算数据显示,当n=7或8时,R_n^{AB}/r、R_n^{CD}/r、R_n^{EF}/r和R_n^{GH}/r分别趋近于各自的极限值。

本研究所运用的优化递推-变换方法,从理论层面突破了传统复杂网络等效分析中对矩阵运算的依赖,丰富了级联型电阻网络的求解方法库,为同类型高维拓扑网络的电特性研究提供了可借鉴的分析框架。在应用层面,本研究所讨论的级联蛛网

电阻网络模型,对于具有蛛网结构材料的电性能以及热性能的分析,具备一定的工程实用价值。

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