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$\tilde{\gamma}_{p-1}h_0g_n$相关谱序列的注记*

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摘要:探究球谱的同伦群是同伦论中的一个重要课题,May谱序列是主要工具。当$p\geqslant 7,n\geqslant 4$时,对于一族可以在Adams谱序列中表示为$\tilde{\gamma}_{p-1}h_0g_n$的永久循环元素,在May谱序列中用线性代数的方法给出相应结论。

关键词:May谱序列;Ext群;Adams谱序列

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Some notes on the spectral sequence relative to $\tilde{\gamma}_{p-1}h_0g_n$*

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Abstract: Detecting the homotopy group of sphere spectrum is an important problem in homotopy theory and the May spectral sequence is the main tool. In the May spectral sequence, some conclusions of the permanent cycles which can be represented by $\tilde{\gamma}_{p-1}h_0g_n$ in the Adams spectral sequence are given with the method of linear algebra, where $p\geqslant 7, n\geqslant 4$.

Keywords: May spectral sequence; Ext group; Adams spectral sequence

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0　引　言

由S^{n+k}到S^n映射的同伦类定义为$\pi_{n+k}(S^n)$,其有着自然的交换群结构并且当$n>k+1$时与n无关,记此群为π_k^s。当$k<0$时,$\pi_k^s=0$;当$k=0$时,$\pi_k^s=\mathbb{Z}$;由文献[1]中Serré有限定理可知,当$k>0$时,π_k^s为有限群,从而这对于研究其素数阶群以及确定π_k^s是有意义的。

近年来,蔡玉梅等[2]证明了当$p\geqslant 7$时,b_1h_1,b_1g_0收敛到同伦群$\pi_*V(1)$中相应非零元;蔡玉梅和王玉玉[3]通过May谱序列研究了当$p\geqslant 11,0\leqslant s\leqslant p-4$时,乘积$b_0g_0\tilde{\alpha}_{s+4}^{(4)}\in \mathrm{Ext}_A^{s+8,q[(s+4)p^3+(s+3)p^2+(s+4)p+(s+3)]+s}(\mathbb{Z}_P,\mathbb{Z}_P)$的非平凡性;Gu等[4]给出了当$p\geqslant 7, s\equiv 0,\pm 1 \bmod(p), n\not\equiv 1 \bmod 3, n>1$时,乘积$\zeta_n\gamma_s\in\pi_*(S)$是非平凡的;王玉玉和刘艳芳[5]指出$\gamma_s\xi_n$在Adams谱序列中由$\tilde{\gamma}_sh_n\in \mathrm{Ext}_A^{s+1,p^nq+sp^2q+(s-1)pq+(s-2)q+s-3}(\mathbb{Z}_P,\mathbb{Z}_P)$表示;$\gamma_{s+3}\sigma_n$在Adams谱序列中由$\tilde{\gamma}_{s+3}\tilde{l}_ng_0\in \mathrm{Ext}_A^{s+8,p^{n+1}q+2p^nq+(s+3)p^2q+(s+3)pq+(s+3)q+s}(\mathbb{Z}_P,\mathbb{Z}_P)$表示

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由王玉玉[6]得到;Liu等[7]研究了元素$\beta_1\omega_n\gamma_s\in\pi_*(S)$在$p>5,n>3,3\leqslant s<p-2$条件下的存在性。特别地,Miller等[8]表明当$s\geqslant 1,p>2$时,$\gamma_s\in\pi_k^S$非平凡;Wang和Zheng[9]给出了γ_s在Adams谱序列中由$\tilde{\gamma}_s\in H^{s,sp^2q+(s-1)pq+(s-2)q+s-3}(A)$表示。

目前对于π_k^S中元素计算的最有力工具为Adams谱序列,其中关键是对模p Steenrod代数A的上同调$\left(H^{*,*}(A)\right)$进行讨论。在A及其对偶A_*中分别存在bar结构和cobar结构,其中包含许多生成元,即便生成元和微分都很清楚的情况下,对于高维Steenrod代数的上同调来讲,也很难计算。因此,对其的决定是一项有意义的工作,其中高维情况的探究可以通过考虑已知元素的乘积来实现。

林金坤[10]给出猜想:当$p\geqslant 7,n\geqslant 3,3\leqslant s<p$时,乘积$\tilde{\gamma}_s h_0g_n$在Adams谱序列中是永久循环且收敛到球面稳定同伦群相应的p阶元素,且在文献[11]中给出$3\leqslant s<p-1$时的证明。本文将文献[11]中滤子的范围扩大至$p-1$时,给出在May谱序列中相应于$\tilde{\gamma}_s h_0g_n$的理论上的准备,并且这也进一步为文献[10]中猜想的完成提供了必要条件。

1　预备知识

前面提到,模p Steenrod代数A的上同调$\left(H^{*,*}(A)\right)$计算是较困难的一项工作,当第一次数是0,1,2,3时,具体有以下结果。$H^{0,*}(A)\cong\mathbb{Z}_p$;$H^{1,*}(A)$由Steenrod代数给出[12];$H^{2,*}(A)$的确定详见文献[13-14],且给出了$H^{2,*}(A)$中的可分解元[13];确定了$H^{3,*}(A)$的情况[15-16]。

目前为止,May[17-18]给出了一个三分次的谱序列,即May谱序列,是计算$H^{*,*}(A)$的有效工具;Ravenel[19]给出了存在收敛于$H^{s,t}(A)$的May谱序列$\left\{E_r^{s,t,*},d_r\right\}$,其中$E_1$项为

$$E_1^{*,*,*}=E\left(h_{i,j}|i>0,j\geqslant 0\right)\otimes P\left(b_{i,j}|i>0,j\geqslant 0\right)\otimes P\left(a_i|i\geqslant 0\right),$$

这里E是外代数,P是多项式代数,并且

$$h_{i,j}\in E_1^{1,2(p^i-1)p^j,2i-1},\ b_{i,j}\in E_1^{2,2(p^i-1)p^{j+1},p(2i-1)},\ a_i\in E_1^{1,2p^i-1,2i+1}。$$

对任意$x\in E_1^{s,t,M}$,定义$\dim x=s$, $\deg x=t$,则有

$$\begin{aligned}&\dim h_{i,j}=\dim a_i=1,\ \dim b_{i,j}=2,\\&\deg h_{i,j}=2\left(p^i-1\right)p^j=\left(p^{i+j-1}+\cdots+p^j\right)q,\\&\deg b_{i,j}=2\left(p^i-1\right)p^{j+1}=\left(p^{i+j}+\cdots+p^{j+1}\right)q,\\&\deg a_i=2p^i-1=\left(p^{i-1}+\cdots+1\right)q+1,\\&M\left(h_{i,j}\right)=2i-1,M\left(b_{i,j}\right)=p(2i-1),M\left(a_i\right)=2i+1,\\&\deg a_0=1,M\left(a_0\right)=1,\ \text{其中}i\geqslant 1,\ j\geqslant 0。\end{aligned}$$

Liu和Wang[20]表明E_1项满足分次交换性

$$\begin{cases}a_mh_{n,j}=h_{n,j}a_m, & h_{m,k}h_{n,j}=-h_{n,j}h_{m,k},\\a_mb_{n,j}=b_{n,j}a_m, & h_{m,k}b_{n,j}=b_{n,j}h_{m,k},\\a_ma_n=a_na_m, & b_{m,n}b_{i,j}=b_{i,j}b_{m,n}。\end{cases}$$

E_1项的分次交换性为计算Ext群中非平凡性元素提供了便利,Zhong和Wang[21]给出了具体求解$E_1^{s,t,*}$中元素的步骤。

2　主要结论及证明

命题2.1　当$p\geqslant 7$, $n\geqslant 4$时,在May谱序列中$E_1^{p+1,t,*}$是一个$\mathbb{Z}_p$模,由$g_1,g_2,\cdots,g_{57}$(具体元素见证明过程)生成,其中$t=q\left[p^{n+1}+2p^n+(p-1)p^2+(p-2)p+p-2\right]+p-4$。

证明　首先考虑$F_1^{p,t,*}$中元素$g=x_1\cdots x_by_1\cdots y_m$,其中$x_i=a_{k_i}$,$y_i=h_{i_w,j_w}$,$k_m\leqslant k_{m-1}\leqslant\cdots\leqslant k_1$,$j_1\leqslant j_2\leqslant\cdots\leqslant j_l$,如果$j_w=j_{w+1}$,那么$i_w\cdots i_{w+1}$,其中$b=p-4$,$m\leqslant s+2$。由文献[21]中方法得到次数方程组

$$\begin{cases}x_{1,0}+\cdots+x_{p-4,0}+y_{1,0}+\cdots+y_{l,0}=p-2+k_1p=c_0,\\x_{1,1}+\cdots+x_{p-4,1}+y_{1,1}+\cdots+y_{l,1}=p-2-k_1+k_2p=c_1,\\x_{1,2}+\cdots+x_{p-4,2}+y_{1,2}+\cdots+y_{l,2}=p-1-k_2+k_3p=c_2,\\x_{1,3}+\cdots+x_{p-4,3}+y_{1,3}+\cdots+y_{l,3}=0-k_3+k_4p=c_3,\\\cdots\cdots\\x_{1,n-1}+\cdots+x_{p-4,n-1}+y_{1,n-1}+\cdots+y_{l,n-1}=0-k_{n-1}+k_np=c_{n-1},\\x_{1,n}+\cdots+x_{p-4,n}+y_{1,n}+\cdots+y_{l,n}=2-k_n+k_{n+1}p=c_n,\\x_{1,n+1}+\cdots+x_{p-4,n+1}+y_{1,n+1}+\cdots+y_{l,n+1}=1-k_{n+1}=c_{n+1}。\end{cases}$$

$\boldsymbol{k}$所有可能的取值为

$$\boldsymbol{k}'=(0,0,\cdots,0);\boldsymbol{k}''=(0,0,0,1,1,\cdots,1,0)。$$

情形1　当$\boldsymbol{k}'=(0,0,\cdots,0)$时,$c=(p-2,p-2,p-1,0,\cdots,0,2,1)$,$\tilde{m}=5$,有次数矩阵

$$\left(\begin{array}{ccccc|cccc|c}1&1&1&\cdots&1&1&1&0&0&0&p-2\\1&1&1&\cdots&1&1&1&0&0&0&p-2\\1&1&1&\cdots&1&1&1&1&0&0&p-1\\0&0&0&\cdots&0&0&0&0&0&0&0\\\vdots&\vdots&\vdots&&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots\\0&0&0&\cdots&0&0&0&0&0&0&0\\0&0&0&\cdots&0&0&0&0&1&1&2\\0&0&0&\cdots&0&0&0&0&1&0&1\end{array}\right),$$

此时得到$E_1^{p+1,t,*}$中的元素$a_3^{p-4}h_{3,0}^2h_{1,2}h_{2,n}h_{1,n}=0$。

情形2　当$\boldsymbol{k}''=(0,0,0,1,1,\cdots,1,0)$时,相对应的$c=(p-2,p-2,p-1,p,p-1,\cdots,p-1,1,1)$,$\widetilde{m}=4<5$,次数矩阵的前4行为

$$\left(\begin{array}{ccccc|cccc|c}1&1&\cdots&1&1&1&1&0&0&p-2\\1&1&\cdots&1&1&1&1&0&0&p-2\\1&1&\cdots&1&1&1&1&1&0&p-1\\1&1&\cdots&1&1&1&1&1&1&p\end{array}\right),$$

次数矩阵第5行可能的情况为

$$\left(\begin{array}{ccccc|cccc|c}1&1&\cdots&1&1&1&1&0&0&p-2\\1&1&\cdots&1&1&1&1&0&0&p-2\\1&1&\cdots&1&1&1&1&1&0&p-1\\1&1&\cdots&1&1&1&1&1&1&p\\\hline1&1&\cdots&1&1&1&1&1&0&p-1\quad(1)\\1&1&\cdots&1&1&0&1&1&1&p-1\quad(2)\\1&1&\cdots&1&1&1&1&0&1&p-1\quad(3)\\1&1&\cdots&1&0&1&1&1&1&p-1\quad(4)\end{array}\right)。$$

选择(1)作为第5行时,次数矩阵的第n+1行为

$$\left(\begin{array}{ccccc|cccc|c}1&1&1&\cdots&1&1&1&1&0&0&p-2\\1&1&1&\cdots&1&1&1&1&0&0&p-2\\1&1&1&\cdots&1&1&1&1&1&0&p-1\\1&1&1&\cdots&1&1&1&1&1&1&p\\1&1&1&\cdots&1&1&1&1&1&0&p-1\\\vdots&\vdots&\vdots&&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots\\1&1&1&\cdots&1&1&1&1&1&0&p-1\\\hline1&0&0&\cdots&0&0&0&0&0&0&1\quad(5)\\0&0&0&\cdots&0&0&1&0&0&0&1\quad(6)\\0&0&0&\cdots&0&0&0&0&1&0&1\quad(7)\end{array}\right),$$

由文献[21]中的方法可知,次数矩阵的第n+2行与第n+1行的情况保持一致,得到$F_1^{p,t,*}$中的元素为

$$\left\{\begin{array}{l}a_{n+2}a_n^{p-5}h_{n,0}h_{n,0}h_{n-2,2}h_{1,3}\\a_n^{p-4}h_{n+2,0}h_{n,0}h_{n-2,2}h_{1,3}\\a_n^{p-4}h_{n,0}h_{n,0}h_{n,2}h_{1,3}\end{array}\right\}\in F_1^{p,t,*}。$$

通过分解或替换,可得E_1^{p+1,t,M_4}中的元素为

$$\left\{\begin{array}{l}g_1=a_n^{p-4}h_{n+2,0}h_{n,0}h_{n-2-i,2+i}h_{i,2}h_{1,3},\\\quad(1\leqslant i\leqslant n-3)\\g_2=a_n^{p-4}h_{n+2,0}h_{n-i,i}h_{i,0}h_{n-2,2}h_{1,3},\\\quad(1\leqslant i\leqslant n-1)\\g_3=a_n^{p-4}h_{n+2-i,i}h_{i,0}h_{n,0}h_{n-2,2}h_{1,3},\\\quad(1\leqslant i\leqslant n+1,i\neq n-2)\\g_4=a_n^{p-5}a_{n-i}h_{i,n-i}h_{n+2,0}h_{n,0}h_{n-2,2}h_{1,3},\\\quad(1\leqslant i\leqslant n-1,i\neq n-2)\\g_5=a_n^{p-4}h_{n,0}h_{n-i,i}h_{i,0}h_{n,2}h_{1,3},\\\quad(1\leqslant i\leqslant n-1)\\g_6=a_{n+2}a_n^{p-5}h_{n-i,i}h_{i,0}h_{n,0}h_{n-2,2}h_{1,3},\\\quad(1\leqslant i\leqslant n-1,i\neq 2)\end{array}\right\}\in E_1^{p+1,t,2n(p-1)+p-7},$$

$g_7=a_n^{p-4}h_{n+2,0}h_{n,0}h_{n-2,2}b_{1,2}\in E_1^{p+1,t,2n(p-1)+2p-7}$,

$g_8=a_n^{p-4}h_{n+2,0}h_{n,0}b_{n-2,1}h_{1,3}\in E_1^{p+1,t,4n(p-1)-4p-1}$。

在选择(2)作为第5行时,得到的次数矩阵的第$n+1$行为

$$\left(\begin{array}{ccccc|cccc|c}1&1&1&\cdots&1&1&1&1&0&0&p-2\\1&1&1&\cdots&1&1&1&1&0&0&p-2\\1&1&1&\cdots&1&1&1&1&1&0&p-1\\1&1&1&\cdots&1&1&1&1&1&1&p\\1&1&1&\cdots&1&1&0&1&1&1&p-1\\\vdots&\vdots&\vdots&&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots\\1&1&1&\cdots&1&1&0&1&1&1&p-1\\\hline1&0&0&\cdots&0&0&0&0&0&0&1\quad(8)\\0&0&0&\cdots&0&0&0&1&0&0&1\quad(9)\\0&0&0&\cdots&0&0&0&0&1&0&1\quad(10)\\0&0&0&\cdots&0&0&0&0&0&1&1\quad(11)\end{array}\right),$$

由文献[21]中的方法,次数矩阵的第$n+2$行与第$n+1$行的情况保持一致,得到$F_1^{p,t,*}$中的元素为

$$\left\{\begin{array}{ll}a_{n+2}a_n^{p-5}h_{4,0}h_{n,0}h_{n-2,2}h_{n-3,3}&a_n^{p-4}h_{4,0}h_{n+2,0}h_{n-2,2}h_{n-3,3}\\a_n^{p-4}h_{4,0}h_{n,0}h_{n,2}h_{n-3,3}&a_n^{p-4}h_{4,0}h_{n,0}h_{n-2,2}h_{n-1,3}\end{array}\right\}\in F_1^{p,t,*}。$$

通过分解或替换,可得$E_1^{p+1,t,2n(p-1)+p-7}$中的元素为:

$$g_9=a_{n+2}a_n^{p-6}a_{n-i}h_{i,n-i}h_{4,0}h_{n,0}h_{n-2,2}h_{n-3,3},\quad(1\leqslant i\leqslant n-1),$$

$$g_{10}=a_n^{p-5}a_{n+2-i}h_{i,n+2-i}h_{n,0}h_{4,0}h_{n-2,2}h_{n-3,3},\quad(1\leqslant i\leqslant n+1),$$

$$g_{11}=a_{n+2}a_n^{p-5}h_{n-i,i}h_{i,0}h_{4,0}h_{n-2,2}h_{n-3,3},\quad(1\leqslant i\leqslant n-1),$$

$$g_{12}=a_{n+2}a_n^{p-5}h_{n,0}h_{4-i,i}h_{i,0}h_{n-2,2}h_{n-3,3},\quad(1\leqslant i\leqslant 3),$$

$$g_{13}=a_{n+2}a_n^{p-5}h_{n,0}h_{4,0}h_{n-2-i,2+i}h_{i,2}h_{n-3,3},\quad(2\leqslant i\leqslant n-3),$$

$$g_{14}=a_n^{p-5}a_{n+2}h_{n,0}h_{4,0}h_{n-2,2}h_{n-3-i,3+i}h_{i,3},\quad(1\leqslant i\leqslant n-4),$$

$$g_{15}=a_n^{p-5}a_{n-i}h_{i,n-i}h_{4,0}h_{n+2,0}h_{n-2,2}h_{n-3,3},\ (1\leqslant i\leqslant n, i\neq n-3, n-2),$$
$$g_{16}=a_n^{p-4}h_{4-i,i}h_{i,0}h_{n+2,0}h_{n-2,2}h_{n-3,3},\ (1\leqslant i\leqslant 3),$$
$$g_{17}=a_n^{p-4}h_{4,0}h_{n+2-i,i}h_{i,0}h_{n-2,2}h_{n-3,3},\ (1\leqslant i\leqslant n+1, i\neq 4),$$
$$g_{18}=a_n^{p-4}h_{4,0}h_{n+2,0}h_{n-2-i,2+i}h_{i,2}h_{n-3,3},\ (2\leqslant i\leqslant n-3),$$
$$g_{19}=a_n^{p-4}h_{4,0}h_{n+2,0}h_{n-2,2}h_{n-3-i,3+i}h_{i,3},\ (1\leqslant i\leqslant n-4),$$
$$g_{20}=a_n^{p-5}a_{n-i}h_{i,n-i}h_{4,0}h_{n,0}h_{n,2}h_{n-3,3},\ (1\leqslant i\leqslant n-1, i\neq n-3, n-2),$$
$$g_{21}=a_n^{p-4}h_{4-i,i}h_{i,0}h_{n,0}h_{n,2}h_{n-3,3},\ (1\leqslant i\leqslant 3),$$
$$g_{22}=a_n^{p-4}h_{4,0}h_{n-i,i}h_{i,0}h_{n,2}h_{n-3,3},\ (1\leqslant i\leqslant n-1, i\neq 3),$$
$$g_{23}=a_n^{p-4}h_{4,0}h_{n,0}h_{n-i,2+i}h_{i,2}h_{n-3,3},\ (1\leqslant i\leqslant n-1),$$
$$g_{24}=a_n^{p-5}a_{n-i}h_{i,n-i}h_{4,0}h_{n,0}h_{n-2,2}h_{n-1,3},\ (1\leqslant i\leqslant n-1, i\neq n-2),$$
$$g_{25}=a_n^{p-4}h_{4-i,i}h_{i,0}h_{n,0}h_{n-2,2}h_{n-1,3},\ (1\leqslant i\leqslant 3),$$
$$g_{26}=a_n^{p-4}h_{4,0}h_{n-i,i}h_{i,0}h_{n-2,2}h_{n-1,3},\ (1\leqslant i\leqslant n-1, i\neq 2, 4),$$
$$g_{27}=a_n^{p-4}h_{4,0}h_{n,0}h_{n-2-i,2+i}h_{i,2}h_{n-1,3},\ (1\leqslant i\leqslant n-3),$$
$$g_{28}=a_n^{p-4}h_{4,0}h_{n,0}h_{n-2,2}h_{n-1-i,3+i}h_{i,3},\ (1\leqslant i\leqslant n-2),$$
$$g_{29}=a_n^{p-4}h_{4,0}h_{n,0}h_{n,2}h_{n-3-i,3+i}h_{i,3},\ (1\leqslant i\leqslant n-4)。$$

以及可以得到以下E_1^{p+1,t,M_4}中的元素:

$$\left\{\begin{array}{l}g_{30}=a_{n+2}a_n^{p-5}h_{4,0}h_{n,0}h_{n-3,3}b_{n-2,1}\\ g_{31}=a_n^{p-4}h_{4,0}h_{n+2,0}h_{n-3,3}b_{n-2,1}\\ g_{32}=a_n^{p-4}h_{4,0}h_{n,0}h_{n-1,3}b_{n-2,1}\end{array}\right\}\in E_1^{p+1,t,4n(p-1)-4p-1},$$

$$\left\{\begin{array}{l}g_{33}=a_n^{p-4}h_{4,0}h_{n+2,0}h_{n-2,2}b_{n-3,2}\\ g_{34}=a_n^{p-4}h_{4,0}h_{n,0}h_{n,2}b_{n-3,2}\\ g_{35}=a_{n+2}a_n^{p-5}h_{n,0}h_{4,0}h_{n-2,2}b_{n-3,2}\end{array}\right\}\in E_1^{p+1,t,4n(p-1)-6p+1},$$

$$g_{36}=a_n^{p-4}h_{n,0}h_{4,0}h_{n-3,3}b_{n,1}\in E_1^{p+1,t,4n(p-1)-5},$$
$$g_{37}=a_n^{p-4}h_{4,0}h_{n,0}h_{n-2,2}b_{n-1,2}\in E_1^{p+1,t,4n(p-1)-2p-3}。$$

在选择(3)作为第5行时,得到的次数矩阵的第$n+1$行为

$$\left(\begin{array}{cccccc|cccc|c}1&1&1&\cdots&1&1&1&1&0&0&p-2\\1&1&1&\cdots&1&1&1&1&0&0&p-2\\1&1&1&\cdots&1&1&1&1&1&0&p-1\\1&1&1&\cdots&1&1&1&1&1&1&p\\1&1&1&\cdots&1&1&1&1&0&1&p-1\\\vdots&\vdots&\vdots&&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots\\1&1&1&\cdots&1&1&1&1&0&1&p-1\\\hline1&0&0&\cdots&0&0&0&0&0&0&1\\0&0&0&\cdots&0&0&0&1&0&0&1\\0&0&0&\cdots&0&0&0&0&0&1&1\end{array}\right),\quad(12)\ (13)\ (14)$$

由文献[21]中的方法,次数矩阵的第$n+2$行与第$n+1$行的情况保持一致,得到$F_1^{p,t,*}$中的元素为

$$\left\{\begin{array}{cc}a_{n+2}a_n^{p-5}h_{n,0}h_{n,0}h_{2,2}h_{n-3,3}&a_n^{p-4}h_{n+2,0}h_{n,0}h_{2,2}h_{n-3,3}\\&a_n^{p-4}h_{n,0}h_{n,0}h_{2,2}h_{n-1,3}\end{array}\right\}\in F_1^{p,t,*}。$$

通过分解或替换,可得E_1^{p+1,t,M_4}中的元素:

$$\left\{\begin{array}{l}g_{38}=a_{n+2}a_n^{p-5}h_{n-i,i}h_{i,0}h_{n,0}h_{2,2}h_{n-3,3},\ (1\leqslant i\leqslant n-1, i\neq 3)\\ g_{39}=a_n^{p-4}h_{n-i,i}h_{i,0}h_{n,0}h_{2,2}h_{n-1,3},\ (1\leqslant i\leqslant n-1)\\ g_{40}=a_n^{p-5}a_{n-i}h_{i,n-i}h_{n+2,0}h_{n,0}h_{2,2}h_{n-3,3},\ (1\leqslant i\leqslant n-1)\\ g_{41}=a_n^{p-4}h_{n+2-i,i}h_{i,0}h_{n,0}h_{2,2}h_{n-3,3},\ (1\leqslant i\leqslant n+1, i\neq n)\\ g_{42}=a_n^{p-4}h_{n+2,0}h_{n,0}h_{1,3}h_{1,2}h_{n-3,3}\\ g_{43}=a_n^{p-4}h_{n+2,0}h_{n,0}h_{2,2}h_{n-3-i,3+i}h_{i,3},\ (1\leqslant i\leqslant n-4)\\ g_{44}=a_n^{p-4}h_{n+2,0}h_{n-i,i}h_{i,0}h_{2,2}h_{n-3,3},\ (1\leqslant i\leqslant n-1)\end{array}\right\}\in E_1^{p+1,t,2n(p-1)+p-7},$$

$$g_{45}=a_n^{p-4}h_{n+2,0}h_{n,0}h_{2,2}b_{n-3,2}\in E_1^{p+1,t,4n(p-1)-6p+1},$$
$$g_{46}=a_n^{p-4}h_{n+2,0}h_{n,0}h_{n-3,3}b_{2,1}\in E_1^{p+1,t,2n(p-1)+4p-9}。$$

在选择(4)作为第5行时,得到的次数矩阵的第$n+1$行:

$$\left(\begin{array}{cccccc|cccc|c}1&1&1&\cdots&1&1&1&1&0&0&p-2\\1&1&1&\cdots&1&1&1&1&0&0&p-2\\1&1&1&\cdots&1&1&1&1&1&0&p-1\\1&1&1&\cdots&1&1&1&1&1&1&p\\1&1&1&\cdots&1&0&1&1&1&1&p-1\\\vdots&\vdots&\vdots&&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots\\1&1&1&\cdots&1&0&1&1&1&1&p-1\\\hline1&0&0&\cdots&0&0&0&0&0&0&1\\0&0&0&\cdots&0&0&1&0&0&0&1\\0&0&0&\cdots&0&0&0&0&1&0&1\\0&0&0&\cdots&0&0&0&0&0&1&1\end{array}\right),\quad(15)\ (16)\ (17)\ (18)$$

由文献[21]中的方法可知,次数矩阵的第$n+2$行与第$n+1$行的情况保持一致,得到$F_1^{p,t,*}$中的元素为

$$\left\{\begin{array}{cc}a_{n+2}a_n^{p-6}a_4h_{n,0}h_{n,0}h_{n-2,2}h_{n-3,3}&a_n^{p-5}a_4h_{n+2,0}h_{n,0}h_{n-2,2}h_{n-3,3}\\a_n^{p-5}a_4h_{n,0}h_{n,0}h_{n,2}h_{n-3,3}&a_n^{p-5}a_4h_{n,0}h_{n,0}h_{n-2,2}h_{n-1,3}\end{array}\right\}\in F_1^{p,t,*}。$$

通过分解或替换,可得E_1^{p+1,t,M_4}中的元素为:

$$\left\{\begin{array}{l} g_{47}=a_{n+2}a_n^{p-6}a_4h_{n-i,i}h_{i,0}h_{n,0}h_{n-2,2}h_{n-3,3},\\ \quad (1\leqslant i\leqslant n-1,\ i\neq 2,3)\\ g_{48}=a_n^{p-5}a_4h_{n-i,i}h_{i,0}h_{n,0}h_{n-2,2}h_{n-1,3},\\ \quad (1\leqslant i\leqslant n-1,\ i\neq 2)\\ g_{49}=a_n^{p-6}a_4a_{n-i}h_{i,n-i}h_{n+2,0}h_{n,0}h_{n-2,2}h_{n-3,3},\\ \quad (1\leqslant i\leqslant n-1,\ i\neq n-3,n-2)\\ g_{50}=a_n^{p-5}a_4h_{n+2-i,i}h_{i,0}h_{n,0}h_{n-2,2}h_{n-3,3},\\ \quad (1\leqslant i\leqslant n+1,\ i\neq n)\\ g_{51}=a_n^{p-5}a_4h_{n+2,0}h_{n-i,i}h_{i,0}h_{n-2,2}h_{n-3,3},\\ \quad (1\leqslant i\leqslant n-1,\ i\neq 2,3)\\ g_{52}=a_n^{p-5}a_4h_{n+2,0}h_{n,0}h_{n-2-i,2+i}h_{i,2}h_{n-3,3},\\ \quad (2\leqslant i\leqslant n-3)\\ g_{53}=a_n^{p-5}a_4h_{n+2,0}h_{n,0}h_{n-2,0}h_{n-3-i,3+i}h_{i,3},\\ \quad (1\leqslant i\leqslant n-4)\\ g_{54}=a_n^{p-5}a_{4-i}h_{i,4-i}h_{n+2,0}h_{n,0}h_{n-2,2}h_{n-3,3},\\ \quad (1\leqslant i\leqslant 4)\\ g_{55}=a_n^{p-5}a_4h_{n-i,i}h_{i,0}h_{n,0}h_{n,2}h_{n-3,3},\\ \quad (1\leqslant i\leqslant n-1,\ i\neq 3) \end{array}\right\}\in E_1^{p+1,\,t,\,2n(p-1)+p-7},$$

$g_{56}=a_n^{p-5}a_4h_{n+2,0}h_{n,0}h_{n-3,3}b_{n-2,1}\in E_1^{p+1,\,t,\,4n(p-1)-4p-1}$,

$g_{57}=a_n^{p-5}a_4h_{n+2,0}h_{n,0}h_{n-2,2}b_{n-3,2}\in E_1^{p+1,\,t,\,4n(p-1)-6p+1}$。

命题 2.2　当 $p\geqslant 7$, $n\geqslant 4$ 时，元素 $\tilde{\gamma}_{p-1}h_0g_n\in H_A^{p+2,t}\left(\mathbb{Z}_p,\mathbb{Z}_p\right)$ 可以被 May 谱序列中的元素 $a_3^{p-4}h_{3,0}h_{2,1}h_{1,2}h_{1,0}h_{1,n}h_{2,n}$ 表示，其中 $t=q\left[p^{n+1}+2p^n+(p-1)p^2+(p-2)p+p-2\right]+p-4$。

证明　由于次数原因，$h_{1,0}h_{1,n}h_{2,n}$ 在May谱序列里是永久循环且非平凡的收敛到 h_0g_n。由文献[9]可知，$a_3^{p-4}h_{3,0}h_{2,1}h_{1,2}$ 在May谱序列里是永久循环且非平凡的收敛到 $\tilde{\gamma}_{p-1}$。

因此，$a_3^{p-4}h_{3,0}h_{2,1}h_{1,2}h_{1,0}h_{1,n}h_{2,n}\in E_1^{p+2,t,*}$ 在May谱序列里是永久循环且非平凡收敛到 $\tilde{\gamma}_{p-1}h_0g_n\in H_A^{p+2,t}\left(\mathbb{Z}_p,\mathbb{Z}_p\right)$。

注　讨论 $\tilde{\gamma}_{p-1}h_0g_n$ 在Adams谱序列中的非平凡性的时候，本文中的结论是解决问题的根本。若 $\tilde{\gamma}_{p-1}h_0g_n$ 非平凡，则以下3个乘积元素 $\tilde{\gamma}_{p-1}h_0$, $\tilde{\gamma}_{p-1}g_n$, h_0g_n 在命题条件下亦是非平凡的。

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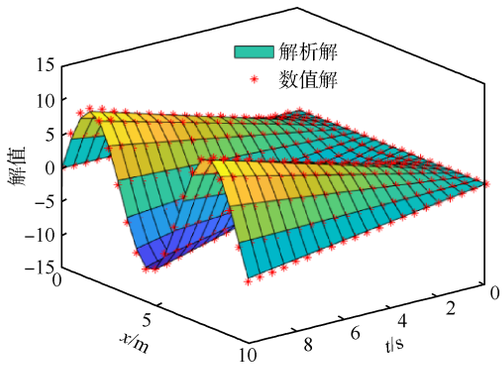


图6　波动方程初值问题对比

4　结束语

本文基于定积分的复化辛普森公式推导出二重积分的双辛普森递推公式,并给出了双辛普森递推算法实现的技术,得出了双辛普森递推算法的收敛速度比双梯形递推算法更快且其内存消耗更小的结论,同时在相同节点分裂次数下,双辛普森递推算法的舍入误差继承了上次分裂结果的1/4,因此大大提高了双辛普森递推算法的计算精度。由于其收敛速度快的原因,双辛普森递推算法计算时程序运行时间更短,这更能体现出双辛普森递推算法的优势所在。

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