DOI：10.19789/j.1004-9398.2025.03.002
文献引用：姜莲霞，张四保．一类广义Euler函数$\varphi_e(n)$的计算公式[J]．首都师范大学学报（自然科学版），2025，46（3）：10-15. JIANG L X，ZHANG S B. Calculation formula of a class of generalized Euler function $\varphi_e(n)$[J]. Journal of Capital Normal University（Natural Science Edition），2025，46（3）：10-15.

一类广义Euler函数$\varphi_e(n)$的计算公式*

姜莲霞**，张四保

（喀什大学数学与统计学院，新疆 喀什　844008）

摘要：令$\varphi_e(n)$为广义Euler函数，其中e与n都是正整数。在有关广义Euler函数$\varphi_e(n)$的性质研究中，对于不同e值广义Euler函数$\varphi_e(n)$的计算公式是至关重要的研究内容之一。针对广义Euler函数$\varphi_e(n)$的计算公式的问题，主要讨论了广义Euler函数$\varphi_{11}(n)$的精确计算公式问题，给出了当n为某些正整数时$\varphi_{11}(n)$的准确计算公式。为检验所给出的计算公式的准确性，将结合Python编程来进行检验。

关键词：广义Euler函数$\varphi_{11}(n)$；莫比乌斯函数$\mu(n)$；计算公式
中图分类号：O156　　**文献标志码**：A

Calculation formula of a class of generalized Euler function $\varphi_e(n)$*

JIANG Lianxia**，ZHANG Sibao

（School of Mathematics and Statistics，Kashi University，Kashi Xinjiang　844008）

Abstract：Let $\varphi_e(n)$ be generalized Euler function，where e and n are positive integers. The calculation formula of generalized Euler function $\varphi_e(n)$ for different values of e is one of the most important contents in the study of the properties of generalized Euler function $\varphi_e(n)$. Aiming at the problem of the calculation formula of the generalized Euler function $\varphi_e(n)$，the accurate calculation formula of the generalized Euler function $\varphi_{11}(n)$ was discussed mainly，and he accurate calculation formula of the generalized Euler function $\varphi_{11}(n)$ was given for n takes some positive integers. In order to check the accuracy of the given calculation formula，it will be checked in combination with Python programming in the article.

Keywords：generalized Euler function $\varphi_{11}(n)$；Möbius function $\mu(n)$；calculation formula
CLC：O156　　**DC**：A

0　序　言

令$\varphi_e(n)=\sum\limits_{\substack{i=1\\(i,n)=1}}^{\left[\frac{n}{e}\right]}1$[1]为广义Euler函数，其是由Cai[2]在2002年引入的一个数论函数。广义Euler函数$\varphi_e(n)$一经提出就引起一些数论爱好者极大的兴趣，解决了一些与广义Euler函数$\varphi_e(n)$有关的方程可解性问题。文献[3]讨论了$k\varphi(Y)=\varphi_2(Y)+$

收稿日期：2022-10-30
*新疆维吾尔自治区高校基本科研业务费科研项目（XJEDU2022P088）
**通信作者：lxjiang_ks@126.com

$S(Y^8)$的可解性,给出该方程有整数解时的k值以及对应的整数解,其中$S(n)$为Smarandache函数;文献[4]解决了方程$\varphi_3(n)=S(n)$的可解性,得到了该方程只有n=49、98共2个整数解;文献[5]解决了方程$\varphi_2(\varphi_3(n))=3^{\omega(n)}$的可解性,得到了该方程的全部20个整数解;文献[6]解决了方程$\varphi_6(n)=\frac{n}{d}$的可解性;文献[7]解决了方程$\varphi_2(m)=2^{\omega(n)}3^{\Omega(n)}$的可解性;文献[8]讨论了包含广义Euler函数$\varphi_2(n)$与$\varphi_6(n)$的一个方程的整数解,给出了方程$\varphi_2(\varphi_6(n))=2^{\omega(n)}$的92个正整数解;文献[9]讨论了当$e$=3,4,6时方程$Z(SL(n))=\varphi_e(n)$的整数解,其中函数$Z(n)$为伪Smarandache函数,函数$SL(n)$为Smarandache最小公倍数函数(Smarandache least common multiple,Smarandache LCM);文献[10]讨论了形如$k\varphi(n)=\varphi_2(n)+S(n^m)$的2个方程的可解性问题,给出了方程$3\varphi(n)=\varphi_2(n)+S(n^{30})$无正整数解以及方程$2\varphi(n)=\varphi_2(n)+S(n^{28})$只有4个正整数解的结论;文献[11]讨论了方程$m\varphi(n)=\varphi_2(n)+S(n^{10})$的可解性问题,给出了方程$m\varphi(n)=\varphi_2(n)+S(n^{10})$只在当$m$=1,2,4,6,8,11,13时有整数解,并给出了对应方程的所有整数解;文献[12]讨论了关于广义Euler函数$\varphi_3(n)$的方程$\varphi(xy)=k(\varphi_3(x)+\varphi_3(y))$的可解性。

对于包含广义Euler函数$\varphi_e(n)$方程可解性的讨论,都是利用$\varphi_e(n)$的计算公式,因而有关函数$\varphi_e(n)$的计算公式成为了广义Euler函数$\varphi_e(n)$研究中最重要的问题。而对于函数$\varphi_e(n)$的计算公式也有一些研究成果。文献[13-15]给出了$\varphi_e(n)$(e=2,3,4,6)的准确计算公式;文献[16]利用初等的方法和技巧给出了部分正整数n的$\varphi_5(n)$准确计算公式;文献[17]讨论了$\varphi_7(n)$的准确计算公式问题,给出了某些特殊类型整数n的$\varphi_7(n)$准确计算公式;文献[18]讨论当$e=pq$、$e=p$与$e=p^2$时广义Euler函数$\varphi_e(n)$的准确计算公式,其中p与q为互不相同的素数;文献[19]讨论了当e为正整数n的特殊正因数时广义Euler函数$\varphi_e(n)$的准确计算公式;文献[20]讨论了当$e=p^r$, $\prod_{i=1}^{t}q_i$时广义Euler函数$\varphi_e(n)$的准确计算公式。

就有关$\varphi_e(n)$的计算公式问题,本文将解决一些特殊正整数n的广义欧拉函数$\varphi_{11}(n)$的准确计算公式。在$\varphi_e(n)$的计算公式问题上,前期有关文献大都是从理论上加以证明或者检验所给出的计算公式准确性时所给出整数n值较小,当正整数n比较大时为检验本文所给出的计算公式的准确性,本文将结合Python编程来进行检验。

1　主要结论及其证明

根据广义Euler函数$\varphi_e(n)$的定义,可证得$\varphi_e(n)=\sum_{d|n}\mu\left(\frac{n}{d}\right)\left[\frac{d}{e}\right]=\sum_{d|n}^{\mu(\frac{n}{d})}\left[\frac{d}{e}\right]$[9],其中

$$\mu(n)=\begin{cases}1, & n=1,\\ (-1)^t, & n\geqslant 2且\alpha_1=\cdots=\alpha_t=1,\\ 0, & n\geqslant 2且存在\alpha_j>1(1\leqslant j\leqslant t),\end{cases}$$

是莫比乌斯函数,$n=\prod_{j=1}^{t}p_j^{\alpha_j}$,且$\alpha_j\geqslant 0$,$p_j(1\leqslant j\leqslant t)$为不同的素数。

定理1　若正整数$n=11^\gamma$,其中$\gamma\geqslant 2$,则$\varphi_{11}(n)=\frac{\varphi(n)}{11}$。

证明　当$n=11^\gamma$时,则由广义欧拉函数$\varphi_e(n)$与莫比乌斯函数$\mu(n)$之间的关系可得

$$\varphi_{11}(n)=\mu(1)\left[\frac{11^\gamma}{11}\right]+\mu(11)\left[\frac{11^{\gamma-1}}{11}\right]=11^{\gamma-1}-11^{\gamma-2}=\frac{11^\gamma-11^{\gamma-1}}{11}=\frac{\varphi(11^\lambda)}{11}=\frac{\varphi(n)}{11},$$

定理1成立。

定理2　若正整数$n=q^\gamma$,则$\varphi_{11}(n)=\frac{\varphi(n)-l_1+l_2}{11}$,式中:$q\neq 11$且为素数,$\gamma\geqslant 2$为整数;$l_1\equiv q^\gamma\equiv ql_2(\mathrm{mod}11)$,$l_2\equiv q^{\gamma-1}(\mathrm{mod}11)$,$1\leqslant l_1,l_2\leqslant 10$。

证明　当正整数$n=q^\gamma$时,则由广义欧拉函数$\varphi_e(n)$与莫比乌斯函数$\mu(n)$之间的关系可得

$$\varphi_{11}(n)=\sum_{d|n}\mu\left(\frac{n}{d}\right)\left[\frac{d}{11}\right]=\mu(1)\left[\frac{q^\gamma}{11}\right]+\mu(11)\left[\frac{q^{\gamma-1}}{11}\right]=\frac{q^\gamma-l_1}{11}-\frac{q^{\gamma-1}-l_2}{11}=\frac{\varphi(n)-l_1+l_2}{11},$$

式中:$1\leqslant l_1,l_2\leqslant 10$;且$l_1\equiv q^\lambda\equiv ql_2(\mathrm{mod}11)$;$l_2\equiv q^{\gamma-1}(\mathrm{mod}11)$。

定理3　若正整数$n=\prod_{j=1}^{t}q_j^{\gamma_j}$,则$\varphi_{11}(n)=\frac{\varphi(n)}{11}$,式中:$t\geqslant 2$,$\gamma_j\geqslant 1$且为整数;$q_j$为互不相同的素数,且$q_j\equiv 1(\mathrm{mod}11)$,$j=1,2,\cdots,t$。

证明　当 $n=\prod_{j=1}^{t}q_j^{\gamma_j}$，且 $q_j\equiv 1(\mathrm{mod}11)$，$j=1,2,\cdots,t$ 时，则 $\left[\frac{d}{11}\right]=\frac{d-1}{11}$，由广义欧拉函数 $\varphi_e(n)$ 与莫比乌斯函数 $\mu(n)$ 间的关系可得

$$\varphi_{11}(n)=\sum_{d|n}\mu\left(\frac{n}{d}\right)\left[\frac{d}{11}\right]=\sum_{d|n}\mu\left(\frac{n}{d}\right)\frac{d-1}{11}=\sum_{d|n}\mu\left(\frac{n}{d}\right)\frac{d}{11}=\frac{\varphi(n)}{11}。$$

定理4　若素数 $q=11n_1+l$，则 $\varphi_{11}(q)=\frac{q-l}{11}$，式中：$n_1$ 为非负整数，$0\leqslant l\leqslant 10$。

证明　当素数 $q=11n_1+l$ 时，由广义欧拉函数 $\varphi_e(n)$ 与莫比乌斯函数 $\mu(n)$ 之间的关系可得

$$\varphi_{11}(q)=\sum_{d|q}\mu(\frac{q}{d})\left[\frac{d}{11}\right]=\mu(1)\left[\frac{q}{11}\right]+\mu(q)\left[\frac{1}{11}\right]=\frac{q-l}{11}。$$

定理5　设正整数 $n=q_1^{\gamma_1}q_2^{\gamma_2}$，且 $(n,11)=1$，若　$l_1\equiv q_1^{\gamma_1-1}q_2^{\gamma_2}(\mathrm{mod}11)$，　$l_2\equiv q_1^{\gamma_1}q_2^{\gamma_2-1}(\mathrm{mod}11)$，$l_3\equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}\ (\mathrm{mod}11)$，　$l_4\equiv q_1^{\gamma_1}q_2^{\gamma_2}(\mathrm{mod}11)$，　$1\leqslant l_1,l_2,l_3,l_4\leqslant 10$，则

$$\varphi_{11}(n)=\frac{\varphi(n)+l_1+l_2-l_3-l_4}{11}。$$

证明　当 $n=q_1^{\gamma_1}q_2^{\gamma_2}$ 时，由广义欧拉函数 $\varphi_e(n)$ 与莫比乌斯函数 $\mu(n)$ 之间的关系可得

$$\varphi(n)=\sum_{d|q}\mu(\frac{q}{d})\left[\frac{d}{11}\right]=\mu(q_1)\left[\frac{q_1^{\gamma_1-1}q_2^{\gamma_2}}{11}\right]+\mu(q_2)\left[\frac{q_1^{\gamma_1}q_2^{\gamma_2-1}}{11}\right]+\mu(q_1q_2)\left[\frac{q_1^{\gamma_1-1}q_2^{\gamma_2-1}}{11}\right]+\mu(1)\left[\frac{q_1^{\gamma_1}q_2^{\gamma_2}}{11}\right]=-\frac{q_1^{\gamma_1-1}q_2^{\gamma_2}-l_1}{11}-\frac{q_1^{\gamma_1}q_2^{\gamma_2-1}-l_2}{11}+\frac{q_1^{\gamma_1-1}q_2^{\gamma_2-1}-l_3}{11}+\frac{q_1^{\gamma_1}q_2^{\gamma_2}-l_4}{11}=\frac{\varphi(n)+l_1+l_2-l_3-l_4}{11},$$

式中：$l_1\equiv q_1^{\gamma_1-1}q_2^{\gamma_2}(\mathrm{mod}11)$；$l_2\equiv q_1^{\gamma_1}q_2^{\gamma_2-1}(\mathrm{mod}11)$；$l_3\equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}(\mathrm{mod}11)$；　$l_4\equiv q_1^{\gamma_1}q_2^{\gamma_2}(\mathrm{mod}11)$；　$1\leqslant l_1,l_2,l_3,l_4\leqslant 10$。

推论1　当正整数 $n=\prod_{j=1}^{2}q_j^{\gamma_j}$，且存在某个 q_j 使得

$$q_j\equiv 1(\mathrm{mod}11),1\leqslant j\leqslant 2,$$

则 $\varphi_{11}(n)=\frac{\varphi(n)}{11}$，其中 $\gamma_j\geqslant 1$ 且为整数，q_j 为不等于11的互不相同的素数，$j=1,2$。

证明　当 $n=q_1^{\gamma_1}q_2^{\gamma_2}$，不妨假设 q_1 满足 $q_1\equiv 1(\mathrm{mod}11)$，则由定理5可得

$$\varphi_{11}(n)=\frac{\varphi(n)+l_1+l_2-l_3-l_4}{11},$$

式中：$1\leqslant l_j\leqslant 10$，$j=1,2,3,4$；且 $l_1\equiv q_1^{\gamma_1-1}q_2^{\gamma_2}\equiv q_2^{\gamma_2}(\mathrm{mod}11)$；$l_2\equiv q_1^{\gamma_1}q_2^{\gamma_2-1}\equiv q_2^{\gamma_2-1}(\mathrm{mod}11)$；$l_3\equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}\equiv q_2^{\gamma_2-1}(\mathrm{mod}11)$；$l_4\equiv q_1^{\gamma_1}q_2^{\gamma_2}\equiv q_2^{\gamma_2}(\mathrm{mod}11)$。因而 $l_1=l_4$，$l_2=l_3$，进而有

$$\varphi_{11}(n)=\frac{\varphi(n)+l_1+l_2-l_3-l_4}{11}=\frac{\varphi(n)}{11}。$$

定理6　设正整数 $n=q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}$，且 $(n,11)=1$，若 $l_1\equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3}(\mathrm{mod}11)$，$l_2\equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3}(\mathrm{mod}11)$，$l_3\equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3-1}(\mathrm{mod}11)$，　$l_4\equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3}(\mathrm{mod}11)$，$l_5\equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3-1}(\mathrm{mod}11)$，$l_6\equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}(\mathrm{mod}11)$，$l_7\equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}(\mathrm{mod}11)$，$l_8\equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}(\mathrm{mod}11)$，其中 $1\leqslant l_j\leqslant 10$，$j=1,2,\cdots,8$，则

$$\varphi_{11}(n)=\frac{\varphi(n)+l_1+l_2+l_3-l_4-l_5-l_6+l_7-l_8}{11}。$$

证明　当 $n=q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}$，由广义欧拉函数 $\varphi_e(n)$ 与莫比乌斯函数 $\mu(n)$ 之间的关系可得

$$\varphi_{11}(n)=\sum_{d|n}\mu\left(\frac{n}{d}\right)\left[\frac{d}{11}\right]=\mu(q_1)\left[\frac{q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3}}{11}\right]+\mu(q_2)\left[\frac{q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3}}{11}\right]+\mu(q_3)\left[\frac{q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3-1}}{11}\right]+\mu(q_1q_2)\left[\frac{q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3}}{11}\right]+\mu(q_1q_3)\left[\frac{q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3-1}}{11}\right]+\mu(q_2q_3)\left[\frac{q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}}{11}\right]+\mu(q_1q_2q_3)\left[\frac{q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}}{11}\right]+\mu(1)\left[\frac{q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}}{11}\right]=-\frac{q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3}-l_1}{11}-\frac{q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3}-l_2}{11}-\frac{q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3-1}-l_3}{11}+\frac{q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3}-l_4}{11}+\frac{q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3-1}-l_5}{11}+\frac{q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}-l_6}{11}-\frac{q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}-l_7}{11}+\frac{q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}-l_8}{11}=\frac{\varphi(n)+l_1+l_2+l_3-l_4-l_5-l_6+l_7-l_8}{11},$$

式中：$l_1\equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3}(\mathrm{mod}11)$；$l_2\equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3}(\mathrm{mod}11)$；$l_3\equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3-1}(\mathrm{mod}11)$；　$l_4\equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3}\ (\mathrm{mod}11)$；$l_5\equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3-1}(\mathrm{mod}11)$；$l_6\equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}(\mathrm{mod}11)$；$l_7\equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}(\mathrm{mod}11)$；　$l_8\equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}\ (\mathrm{mod}11)$；$1\leqslant l_j\leqslant 10$，$j=1,2,\cdots,8$。

推论2　当正整数 $n=\prod_{j=1}^{3}q_j^{\gamma_j}$，且存在某个 q_j 使得

$$q_j \equiv 1(\mathrm{mod}11), 1 \leqslant j \leqslant 3,$$

则$\varphi_{11}(n)=\dfrac{\varphi(n)}{11}$,其中$\gamma_j \geqslant 1$且为整数,$q_j$为不等于11的互不相同的素数,$j=1,2,3$。

证明　当$n=q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}$,同样不妨假设$q_1$满足$q_1 \equiv 1(\mathrm{mod}11)$,则由定理6可得

$$\varphi_{11}(n)=\frac{\varphi(n)+l_1+l_2+l_3-l_4-l_5-l_6+l_7-l_8}{11},$$

式中:$1 \leqslant l_j \leqslant 10, j=1,2,\cdots,8$;且$l_1 \equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3} \equiv q_2^{\gamma_2}q_3^{\gamma_3}(\mathrm{mod}11)$;　$l_2 \equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3} \equiv q_2^{\gamma_2-1}q_3^{\gamma_3}(\mathrm{mod}11)$;　$l_3 \equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3-1} \equiv q_2^{\gamma_2}q_3^{\gamma_3-1}(\mathrm{mod}11)$;　$l_4 \equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3} \equiv q_2^{\gamma_2-1}q_3^{\gamma_3}(\mathrm{mod}11)$;　$l_5 \equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3-1} \equiv q_2^{\gamma_2}q_3^{\gamma_3-1}(\mathrm{mod}11)$;　$l_6 \equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3-1} \equiv q_2^{\gamma_2}q_3^{\gamma_3}(\mathrm{mod}11)$;　$l_7 \equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3-1} \equiv q_2^{\gamma_2-1}q_3^{\gamma_3-1}(\mathrm{mod}11)$;　$l_8 \equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3} \equiv q_2^{\gamma_2}q_3^{\gamma_3}(\mathrm{mod}11)$。因而$l_1=l_8, l_2=l_4, l_3=l_5, l_6=l_7$,进而有

$$\varphi_{11}(n)=\frac{\varphi(n)+l_1+l_2+l_3-l_4-l_5-l_6+l_7-l_8}{11}=\frac{\varphi(n)}{11}。$$

定理7　正整数$n=q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4}$,且$(n,11)=1$,若$l_1 \equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4}(\mathrm{mod}11)$, $l_2 \equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4}(\mathrm{mod}11)$, $l_3 \equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4}(\mathrm{mod}11)$, $l_4 \equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4-1}(\mathrm{mod}11)$, $l_5 \equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4}(\mathrm{mod}11)$, $l_6 \equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4}(\mathrm{mod}11)$, $l_7 \equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4-1}(\mathrm{mod}11)$, $l_8 \equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4}(\mathrm{mod}11)$, $l_9 \equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4-1}(\mathrm{mod}11)$, $l_{10} \equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4-1}(\mathrm{mod}11)$, $l_{11} \equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4}(\mathrm{mod}11)$, $l_{12} \equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4-1}(\mathrm{mod}11)$, $l_{13} \equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4-1}(\mathrm{mod}11)$, $l_{14} \equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4-1}(\mathrm{mod}11)$, $l_{15} \equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4-1}(\mathrm{mod}11)$, $l_{16} \equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4}(\mathrm{mod}11)$,其中$1 \leqslant l_j \leqslant 10, j=1,2,\cdots,16$,则

$$\varphi_{11}(n)=\frac{\varphi(n)+l_1+l_2+l_3+l_4-l_5-l_6-l_7-l_8-l_9-l_{10}+l_{11}+l_{12}+l_{13}+l_{14}-l_{15}-l_{16}}{11}。$$

证明　当$n=q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4}$时,由广义欧拉函数$\varphi_e(n)$与莫比乌斯函数$\mu(n)$之间的关系可得

$$\varphi_{11}(n)=\sum_{d|n}\mu\left(\frac{n}{d}\right)\left[\frac{d}{11}\right]=\mu(q_1)\left[\frac{q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4}}{11}\right]+\mu(q_2)\left[\frac{q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4}}{11}\right]+\mu(q_3)\left[\frac{q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4}}{11}\right]+$$
$$\mu(q_4)\left[\frac{q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4-1}}{11}\right]+\mu(q_1q_2)\left[\frac{q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4}}{11}\right]+\mu(q_1q_3)\left[\frac{q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4}}{11}\right]+\mu(q_1q_4)\left[\frac{q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4-1}}{11}\right]+$$
$$\mu(q_2q_3)\left[\frac{q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4}}{11}\right]+\mu(q_2q_4)\left[\frac{q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4-1}}{11}\right]+\mu(q_3q_4)\left[\frac{q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4-1}}{11}\right]+$$
$$\mu(q_1q_2q_3)\left[\frac{q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4}}{11}\right]+\mu(q_1q_2q_4)\left[\frac{q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4-1}}{11}\right]+\mu(q_1q_3q_4)\left[\frac{q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4-1}}{11}\right]+$$
$$\mu(q_2q_3q_4)\left[\frac{q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4-1}}{11}\right]+\mu(q_1q_2q_3q_4)\left[\frac{q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4-1}}{11}\right]+\mu(1)\left[\frac{q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4}}{11}\right]=$$
$$-\frac{q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4}-l_1}{11}-\frac{q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4}-l_2}{11}-\frac{q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4}-l_3}{11}-\frac{q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4-1}-l_4}{11}+\frac{q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4}-l_5}{11}+$$
$$\frac{q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4}-l_6}{11}+\frac{q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4-1}-l_7}{11}+\frac{q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4}-l_8}{11}+\frac{q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4-1}-l_9}{11}+\frac{q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4-1}-l_{10}}{11}-$$
$$\frac{q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4}-l_{11}}{11}-\frac{q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4-1}-l_{12}}{11}-\frac{q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4-1}-l_{13}}{11}-$$
$$\frac{q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4-1}-l_{14}}{11}+\frac{q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4-1}-l_{15}}{11}+\frac{q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4}-l_{16}}{11}=$$
$$\frac{\varphi(n)+l_1+l_2+l_3+l_4-l_5-l_6-l_7-l_8-l_9-l_{10}+l_{11}+l_{12}+l_{13}+l_{14}-l_{15}-l_{16}}{11},$$

式中:

$$l_1 \equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4}(\mathrm{mod}11); l_2 \equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4}(\mathrm{mod}11);$$
$$l_3 \equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4}(\mathrm{mod}11); l_4 \equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4-1}(\mathrm{mod}11); l_5 \equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4}(\mathrm{mod}11);$$
$$l_6 \equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4}(\mathrm{mod}11); l_7 \equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4-1}(\mathrm{mod}11); l_8 \equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4}(\mathrm{mod}11);$$
$$l_9 \equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4-1}(\mathrm{mod}11); l_{10} \equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4-1}(\mathrm{mod}11); l_{11} = q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4}(\mathrm{mod}11);$$
$$l_{12} \equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4-1}(\mathrm{mod}11); l_{13} \equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4-1}(\mathrm{mod}11); l_{14} \equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4-1}(\mathrm{mod}11);$$
$$l_{15} \equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4-1}(\mathrm{mod}11); l_{16} \equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4}(\mathrm{mod}11); 1 \leqslant l_j \leqslant 10, j=1,2,\cdots,16。$$

推论3 当正整数$n=\prod_{j=1}^{4}q_j^{\gamma_j}$,且存在某个$q_j$使得

$$q_j\equiv 1(\mathrm{mod}11),1\leqslant j\leqslant 4,$$

则$\varphi_{11}(n)=\dfrac{\varphi(n)}{11}$,其中$\gamma_j\geqslant 1$且为整数,$q_j$为不等于11的互不相同的素数,$j=1,2,3,4$。

证明 当$n=q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4}$,不妨假设$q_1$满足$q_1\equiv 1(\mathrm{mod}11)$,则由定理7可得

$$\varphi_{11}(n)=\frac{\varphi(n)+l_1+l_2+l_3+l_4-l_5-l_6-l_7-l_8-l_9-l_{10}+l_{11}+l_{12}+l_{13}+l_{14}-l_{15}-l_{16}}{11},$$

式中:$1\leqslant l_j\leqslant 10$,$j=1,2,\cdots,16$;且

$$l_1\equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4}\equiv q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4}(\mathrm{mod}11);l_2\equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4}\equiv q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4}(\mathrm{mod}11);$$

$$l_3\equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4}\equiv q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4}(\mathrm{mod}11);l_4\equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4-1}\equiv q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4-1}(\mathrm{mod}11);$$

$$l_5\equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4}\equiv q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4}(\mathrm{mod}11);l_6\equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4}\equiv q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4}(\mathrm{mod}11);$$

$$l_7\equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4-1}\equiv q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4-1}(\mathrm{mod}11);l_8\equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4}\equiv q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4}(\mathrm{mod}11);$$

$$l_9\equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4-1}\equiv q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4-1}(\mathrm{mod}11);l_{10}\equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4-1}\equiv q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4-1}(\mathrm{mod}11);$$

$$l_{11}=q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4}\equiv q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4}(\mathrm{mod}11);l_{12}\equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4-1}\equiv q_2^{\gamma_2-1}q_3^{\gamma_3}q_4^{\gamma_4-1}(\mathrm{mod}11);$$

$$l_{13}\equiv q_1^{\gamma_1-1}q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4-1}\equiv q_2^{\gamma_2}q_3^{\gamma_3-1}q_4^{\gamma_4-1}(\mathrm{mod}11);l_{14}\equiv q_1^{\gamma_1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4-1}\equiv q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4-1}(\mathrm{mod}11);$$

$$l_{15}\equiv q_1^{\gamma_1-1}q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4-1}\equiv q_2^{\gamma_2-1}q_3^{\gamma_3-1}q_4^{\gamma_4-1}(\mathrm{mod}11);l_{16}\equiv q_1^{\gamma_1}q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4}\equiv q_2^{\gamma_2}q_3^{\gamma_3}q_4^{\gamma_4}(\mathrm{mod}11)。$$

则有$l_1=l_{16},l_2=l_5,l_3=l_6,l_4=l_7,l_8=l_{11},l_9=l_{12},l_{10}=l_{13},l_{14}=l_{15}$,进而有

$$\varphi_{11}(n)=\frac{\varphi(n)+l_1+l_2+l_3+l_4-l_5-l_6-l_7-l_8-l_9-l_{10}+l_{11}+l_{12}+l_{13}+l_{14}-l_{15}-l_{16}}{11}=\frac{\varphi(n)}{11}。$$

2 应 用

当n取值比较小时,经计算可以验证推论1、2、3是正确的;当n取值较大时,要检验本文中推论1、2、3,手算笔录具有一定困难,为此可以运用欧拉函数的定义及Python编程来检验推论1、2、3的结论。现先给出其编程程序为:

```
    def gcd(a,b):
  if b==0:
    return a
  else:
    return gcd(b,a%b)
sz=[ ]
for i in range(1,⌊n/11⌋+1):
  index_gcd=gcd(i,n)
  if index_gcd==1:
    sz append(i)
    print(len(sz))。
```

例1 当$n=41^3\times 67^2$时,利用上述编程可求出序列$1,2,\cdots,\left\lfloor\dfrac{41^3\times 67^2}{11}\right\rfloor=28\,126\,033$中与$n=41^3\times 67^2$互素的整数的个数为27 030 480个,即$\varphi_{11}(41^3\times 67^2)=27\,030\,480$,而运用推论1的结论,可以得到

$$\varphi_{11}(41^3\times 67^2)=\frac{\varphi(41^3\times 67^2)}{11}=\frac{41^3\times 67^2\times\left(1-\frac{1}{41}\right)\times\left(1-\frac{1}{67}\right)}{11}=27\,030\,480。$$

例2 当$n=17\times 23^2\times 37^2$时,利用上述编程可求出序列$1,2,\cdots,\left\lfloor\dfrac{17\times 23^2\times 37^2}{11}\right\rfloor=1\,119\,219$中与$n=17\times 23^2\times 37^2$互素的整数的个数为980 352个,即$\varphi_{11}(17\times 23^2\times 37^2)=980\,352$,运用推论2的结论,可以得到

$$\varphi_{11}(17\times 23^2\times 37^2)=\frac{\varphi(17\times 23^2\times 37^2)}{11}=\frac{17\times 23^2\times 37^2\times\left(1-\frac{1}{17}\right)\times\left(1-\frac{1}{23}\right)\times\left(1-\frac{1}{37}\right)}{11}=980\,352。$$

例3 当$n=5^2\times 7^2\times 13^2\times 89$时,利用上述编程可以求出序列$1,2,\cdots,\left\lfloor\dfrac{5^2\times 7^2\times 13^2\times 89}{11}\right\rfloor=1\,675\,020$中与$n=5^2\times 7^2\times 13^2\times 89$互素的整数的个数为1 048 320个,即$\varphi_{11}(5^2\times 7^2\times 13^2\times 89)=1\,048\,320$,运用推论3可得

$$\varphi_{11}(5^2\times 7^2\times 13^2\times 89)=\frac{\varphi(5^2\times 7^2\times 13^2\times 89)}{11}=$$

$$\frac{5^2\times7^2\times13^2\times89\times\left(1-\frac{1}{5}\right)\times\left(1-\frac{1}{7}\right)\times\left(1-\frac{1}{13}\right)\times\left(1-\frac{1}{89}\right)}{11}=1\,048\,320。$$

3　结束语

本文主要讨论了广义Euler函数$\varphi_{11}(n)$的精确计算公式问题,给出了当n为某些正整数时$\varphi_{11}(n)$的准确计算公式。推论1、2、3给出了当n具有2、3、4个互异的素因子,当素因子中有某个素因子q_j满足$q_j\equiv1(\mathrm{mod}11)$时,$\varphi_{11}(n)=\frac{\varphi(n)}{11}$。文献[13-15]给出了$\varphi_e(n)$($e$=2、3、4、6)的准确计算公式,由$\varphi_e(n)$($e$=2、3、4、6)公式可知,当$n=\prod_{j=1}^{s}q_j^{\gamma_j}$且当素因子中有某个素因子$q_j$满足$q_j\equiv1(\mathrm{mod}e)$时,$\varphi_e(n)=\frac{\varphi(n)}{e}$;王容和廖群英[16]讨论了当$n=\prod_{j=1}^{s}q_j^{\gamma_j}$,$s$=2,3且当素因子中有某个素因子$q_j$满足$q_j\equiv1(\mathrm{mod}5)$时,有$\varphi_5(n)=\frac{\varphi(n)}{5}$;周健和张四保[17]讨论了当$n=\prod_{j=1}^{s}q_j^{\gamma_j}$,$s$=2,3且当素因子中有某个素因子$q_j$满足$q_j\equiv1(\mathrm{mod}7)$时,有$\varphi_7(n)=\frac{\varphi(n)}{7}$。故由以上结论,提出如下问题1。

问题1　当$n=\prod_{j=1}^{s}q_j^{\gamma_j}$,$s\geqslant2$,当素因子$q_1,q_2,\cdots,q_s$中有某个素因子$q_j(1\leqslant j\leqslant s)$满足$q_j\equiv1(\mathrm{mod}e)$时,是否都有$\varphi_e(n)=\frac{\varphi(n)}{e}$?

如果该问题1被证明是正确的,那么可以给出当e与n为某些正整数时广义欧拉函数$\varphi_e(n)$的部分值。

参 考 文 献

[1] 董坤,廖群英,张梦蝶.关于方程$\varphi_6(n)=\frac{n}{d}$的一个注记[J].四川师范大学学报(自然科学版),2022,45(4):489-494.

[2] CAI T X. A congruence involving the quotients of Euler and its applications(I)[J]. Acta Aritmetica, 2002, 103(4):313-320.

[3] 张四保,姜莲霞.数论函数方程$k\varphi(Y)=\varphi_2(Y)+S(Y^8)$的解[J].江西师范大学学报(自然科学版),2021,45(2):194-197.

[4] 李昌吉.方程$\varphi_3(n)=S(n)$的可解性[J].数学的实践与认识,2020,50(24):179-188.

[5] 李昌吉.方程$\varphi_2(\varphi_3(n))=3^{\omega(n)}$的可解性[J].数学的实践与认识,2020,50(8):302-306.

[6] 张四保.广义Euler函数方程$\varphi_6(n)=\frac{n}{d}$的可解性[J].西南大学学报(自然科学版),2019,41(12):50-56.

[7] 张四保,姜莲霞.包含广义Euler函数$\varphi_2(m)$的一方程的正整数解[J].广西大学学报(自然科学版),2021,46(1):226-230.

[8] 张四保.包含两个广义Euler函数的一个方程的解[J].吉林大学学报(理学版),2022,60(2):189-201.

[9] 杜珊,廖群英,王慧莉.方程$Z(SL(n))=\varphi_e(n)$(e=3,4,6)的正整数解[J].四川师范大学学报(自然科学版),2022,45(1):21-26.

[10] 姜莲霞,张四保,傅湧.形如$k\varphi(n)=\varphi_2(n)+S(n^m)$的两个方程的可解性[J].首都师范大学学报(自然科学版),2021,42(6):8-11+70.

[11] 郑惠.数论函数方程$m\varphi(n)=\varphi_2(n)+S(n^{10})$的解[J].江西科学,2022,40(2):219-222.

[12] 姜莲霞,张四保.有关广义欧拉函数$\varphi_3(n)$的一方程的解[J].首都师范大学学报(自然科学版),2020,41(6):1-5.

[13] CAI T X, SHEN Z Y, HU M J. On the parity of the generalized Euler function[J]. 数学进展,2013,42(4):505-510.

[14] 丁煜.广义欧拉函数及其性质[D].杭州:浙江大学,2008.

[15] SHEN Z Y, CAI T X, HU M J. On the parity of the generalized Euler function(II)[J]. 数学进展,2016,45(4):509-519.

[16] 王容,廖群英.关于广义欧拉函数$\varphi_5(n)$[J].四川师范大学学报(自然科学版),2018,41(4):445-449.

[17] 周健,张四保.广义Euler函数$\varphi_7(N)$的计算公式[J].数学的实践与认识,2019,49(7):294-298.

[18] LIAO Q Y, LUO W L. The computing formula for two classes generalized Euler functions [J]. Journal of Mathematics(PRC),2019,39(1):97-110.

[19] 廖群英.一类广义欧拉函数的准确计算公式[J].四川师范大学学报(自然科学版),2019,42(3):354-357.

[20] LIAO Q Y. The computing formula for generalized Euler functions [J]. Journal of University of Science and Technology of China,2019,49(8):614-619.

(责任编辑:马田田)