DOI:10.19789/j.1004-9398.2015.06.002

关于拟 Hopf 代数的 Maschke 型定理

杨存絜
(首都师范大学数学科学学院,北京 100048)

摘 要

设 H 是拟 Hopf 代数,本文给出了 Maschke 型定理.

关键词: 拟 Hopf 代数,规范变换,拟二上循环,Maschke 型定理.
中图分类号: O153.3

§1 引 言

F. Panaite 在文[18]中定理 2.3 中给出 Maschke 型定理: 令 H 是拟 Hopf 代数, H 是半单 Artinian 的 $\Leftrightarrow \varepsilon(\int) \neq 0$,在此基础上本文对 Maschke 型定理做进一步研究.

在本节里我们回顾一些定义以及符号. k 是一个域,而且所有的代数、线性空间等都在 k 上考虑,如果不声明, $\otimes$ 都指 $\otimes_k$.

定义 1.1 令 H 是一个 k - 代数, $\Delta: H \to H\otimes H$, $\varepsilon: H \to k$ 是两个代数同态, H 叫做拟双代数(quasi-bialgebra),若存在可逆元 $\varphi \in H\otimes H\otimes H$ 使得 $\forall h \in H$ 有

$$(1.1)\quad (I\otimes\Delta)(\Delta(h)) = \varphi(\Delta\otimes I)(\Delta(h))\varphi^{-1},$$

$$(1.2)\quad (I\otimes I\otimes\Delta)(\varphi)(\Delta\otimes I\otimes I)(\varphi) = (1\otimes\varphi)(I\otimes\Delta\otimes I)(\varphi)(\varphi\otimes 1),$$

$$(1.3)\quad (\varepsilon\otimes I)(\Delta(h)) = 1\otimes h, (I\otimes\varepsilon)(\Delta(h)) = h\otimes 1,$$

$$(1.4)\quad (I\otimes\varepsilon\otimes I)(\varphi) = 1\otimes 1\otimes 1,$$

这里 $I = id_H$,映射 Δ 叫做余乘法(comultiplication), ε 叫做余单位(counit). H 叫做拟 Hopf 代数(quasi-Hopf algebra),若 H 是拟双代数,且存在反代数自同构 $S: H \to H$ 和 $\alpha, \beta \in H$ 使得 $\forall h \in H$ 有

$$(1.5)\quad \sum S(h_1)\alpha h_2 = \varepsilon(h)\alpha, \sum h_1\beta S(h_2) = \varepsilon(h)\beta,$$

$$(1.6)\quad \sum X^1\beta S(X^2)\alpha X^3 = 1, \sum S(x^1)\alpha x^2\beta S(x^3) = 1.$$

这里 $\varphi = \sum X^1\otimes X^2\otimes X^3$, $\varphi^{-1} = \sum x^1\otimes x^2\otimes x^3$,此时 S 叫做 H 的反极(antipode).

进而有

$$(1.7)\quad (\varepsilon\otimes I\otimes I)(\varphi) = 1\otimes 1\otimes 1, (I\otimes I\otimes\varepsilon)(\varphi) = 1\otimes 1\otimes 1.$$

若 $H = (H, \Delta, \varepsilon, \varphi)$ 是拟双代数,我们将 Drinfel'd 结合子(associator) φ 的张量分量用大写的字母表示,例如

$$\varphi = \sum X^1\otimes X^2\otimes X^3 = \sum Y^1\otimes Y^2\otimes Y^3 = \sum Z^1\otimes Z^2\otimes Z^3 = \cdots,$$

而 φ^{-1} 的张量分量用小写的字母表示,例如

$$\varphi^{-1} = \sum x^1\otimes x^2\otimes x^3 = \sum y^1\otimes y^2\otimes y^3 = \sum z^1\otimes z^2\otimes z^3 = \cdots$$

定义 1.2 令 H 是拟双代数,可逆元 $F \in H\otimes H$ 叫做规范变换(gauge transformation),若满足关系

$$(\varepsilon\otimes id)(F) = (id\otimes\varepsilon)(F) = 1.$$

记 $F = \sum F^1\otimes F^2 = \sum f^1\otimes f^2 = \sum \bar{F}^1\otimes\bar{F}^2 = \sum \bar{f}^1\otimes\bar{f}^2 = \sum \underline{F}^1\otimes\underline{F}^2 = \sum \underline{f}^1\otimes\underline{f}^2\cdots$,而且 $F^{-1} = \sum G^1\otimes G^2 = \sum g^1\otimes g^2 = \sum \bar{G}^1\otimes\bar{G}^2 = \sum \bar{g}^1\otimes\bar{g}^2 = \sum \underline{G}^1\otimes\underline{G}^2 = \sum \underline{g}^1\otimes\underline{g}^2\cdots$.

定义 1.3 设 $H = (H, \Delta, \varepsilon, S, \varphi, \alpha, \beta)$ 是拟 Hopf 代数, $F \in H\otimes H$ 是规范变换,若

$$(1\otimes F)(I\otimes\Delta)(F)\varphi = (F\otimes 1)(\Delta\otimes I)(F),$$

收稿日期: 2014-10-29

(C)1994-2020 China Academic Journal Electronic Publishing House. All rights reserved. http://www.cnki.net

称 F 是拟二上循环(quasi two-cocycle) .

定义 1.4　令 H 是拟双代数,A 是 k - 向量空间,称 A 是左 H - 模代数,如果 A 是左 H - 模,A 有乘法和单位元 1_A,满足下面条件:

(a) $(ab)c=\sum (X^1\cdot a)[X^2\cdot b)(X^3\cdot c)]$,

(b) $h\cdot(ab)=\sum (h_1\cdot a)(h_2\cdot b)$,

(c) $h\cdot 1_A=\varepsilon(h)1_A$.

这里 $\forall a,b,c\in A,\forall h\in H,\varphi=\sum X^1\otimes X^2\otimes X^3$,$h\otimes a\to h\cdot a$ 是 A 的 H - 模结构.

定义 1.5　令 H 是拟双代数,A 是左 H - 模代数,A 与 H 的 Smash 积 $A\#H$ 的定义如下:

$$(a\#h)(b\#g)=\sum (x^1\cdot a)(x^2h_1\cdot b)\#x^3h_2g,$$

这里 $\forall a,b\in A,\forall h,g\in H,\varphi^{-1}=\sum x^1\otimes x^2\otimes x^3$.

§2　基本结论

本节先给出一般化的简单结论.

命题 2.1　A 是左 H - 模代数,定义 $(a\#h)\cdot b=a(h\cdot b)$,$\forall (a\#h)\in A\#H,\forall b\in A$,则 A 是左 $A\#H$ - 模.

证明

$$\begin{aligned}
&[(a\#h)(b\#g)]\cdot c\\
=&\sum [(x^1\cdot a)(x^2h_1\cdot b)\#x^3h_2g]\cdot c\\
=&\sum [(x^1\cdot a)(x^2h_1\cdot b)][((x^3h_2g)\cdot c)]\\
=&\sum [(x^1\cdot a)(x^2h_1\cdot b)][x^3\cdot((h_2g)\cdot c)]\\
=&\sum a[(h_1\cdot b)((h_2g)\cdot c)]\\
=&\sum a[h\cdot(b(g\cdot c))]\\
=&(a\#h)\cdot(b(g\cdot c))\\
=&(a\#h)[(b\#g)\cdot c].
\end{aligned}$$

命题 2.2　若 A 是 H - 模代数,则 $A^H\cong End({}_{A\#H}A)$ 是反代数同构.

证明　$\phi:A^H\to End({}_{A\#H}A)$,$a\to R_a$ 其中 R_a 是右乘运算,ϕ 显然是双射. 由于

$$\begin{aligned}
R_{ab}(c)=&c(ab)=\sum [(y^1\cdot c)(y^2\cdot a)](y^3\cdot b)\\
=&\sum [(y^1\cdot c)(\varepsilon(y^2)a)](y^3\cdot b)\\
=&(ca)b=R_bR_a(c)
\end{aligned}$$

从而 ϕ 是反代数同构.

命题 2.3　若 H 是拟双代数,A 是左 H - 模代数,t 是 H 的左积分,满足 $t\cdot c=1$,其中 $c\in A$,则 A 是投射的左 $A\#H$ - 模.

证明　定义 $f:A\#H\to A$,$f(a\#h)=\varepsilon(h)a$. 易证 f 是左 $A\#H$ - 模满同态. 再定义 $g:A\to A\#H$,$g(a)=\sum (x^1\cdot a)(x^2t_1\cdot c)\#(x^3t_2)$. 有

$$\begin{aligned}
fg(a)=&f(\sum (x^1\cdot a)(x^2t_1\cdot c)\#(x^3t_2))\\
=&\sum (x^1\cdot a)(x^2\varepsilon(x^3)\varepsilon(t_2)t_1\cdot c)\\
=&\sum (x^1\cdot a)(x^2\varepsilon(x^3)t\cdot c)=a.
\end{aligned}$$

对任意的 $b\#h\in A\#H$,

$$\begin{aligned}
&(b\#h)\cdot g(a)\\
=&(b\#h)\sum [x^1\cdot a)(x^2t_1\cdot c)\#(x^3t_2)]\\
=&(b\#h)[a\#t)(c\#1)]\\
=&[(b\#h)(a\#t)](c\#1)\\
=&\sum [x^1\cdot b)(x^2h_1\cdot a)\#x^3h_2t](c\#1)\\
=&\sum [x^1\cdot b)(x^2h_1\cdot a)\#\varepsilon(x^3h_2)t](c\#1)\\
=&[b(h\cdot a)\#t](c\#1)\\
=&\sum [x^1\cdot(b(h\cdot a))](x^2t_1\cdot c)\#(x^3t_2)\\
=&g(b(h\cdot a))\\
=&g((b\#h)\cdot a)
\end{aligned}$$

则 f 是分裂左 $A\#H$ - 模同态.

在文 [2] 中对 Hopf 代数情形下的结论:

$$a\#h=\sum (1\#h_2)(S^{-1}(h_1)\cdot a\#1)$$

给出推广: 若 $H=(H,\Delta,\varepsilon,S,\varphi,\alpha,\beta)$ 是拟 Hopf 代数,A 是左 H - 模代数,那么 $\forall h\in H,a,b\in A$,有

$$\begin{aligned}
a\#h=&\sum (1\#x^3h_2)\cdot\\
&(S^{-1}(S(x^1)\alpha x^2h_1X^1\beta S(X^2))\cdot a\#X^3).
\end{aligned}$$

类似的,我们对 Hopf 代数情形下的结论:

(1) $(h\cdot a)b=\sum h_1\cdot(aS(h_2)\cdot b)$,

(2) $a(h\cdot b)=\sum h_2\cdot((S^{-1}(h_1)\cdot a)b)$

给出如下推广

命题 2.4　若 $H=(H,\Delta,\varepsilon,S,\varphi,\alpha,\beta)$ 是拟 Hopf 代数,A 是左 H - 模代数,那么 $\forall h\in H,a,b\in A$,有

(1) $(h\cdot a)b=\sum X^1h_1\cdot((x^1\cdot a)(x^2\beta S(X^2h_2x^3)\alpha X^3)\cdot b)$,

(2) 若 S 是双射,则

$$a(h\cdot b)=\sum x^3h_2\cdot((S^{-1}(S(x^1)\alpha x^2h_1X^1\beta S(X^2))\cdot a)(X^3\cdot b)).$$

证明

$$\begin{aligned}
&(1)\quad \sum X^1h_1\cdot((x^1\cdot a)(x^2\beta S(X^2h_2x^3)\alpha X^3)\cdot b)\\
=&\sum X^1\cdot\{[(h_1)_1x^1\cdot a)]\cdot
\end{aligned}$$

(C)1994-2020 China Academic Journal Electronic Publishing House. All rights reserved.　http://www.cnki.net

$$
\begin{aligned}
&\left[(h_1)_2 x^2 \beta S(X^2 h_2 x^3)\alpha X^3\right]\cdot b\}\\
=&\sum (X_1^1(h_1)_1 x^1 \cdot a)\cdot\\
&\left[X_2^1(h_1)_2 x^2 \beta S(X^2 h_2 x^3)\alpha X^3\right]\cdot b\\
=&\sum (X_1^1 x^1 h_1 \cdot a)(X_2^1 x^2(h_2)_1 \beta S(X^2 x^3(h_2)_2)\alpha X^3)\cdot b\\
=&\sum (X_1^1 x^1 h_1 \cdot a)(X_2^1 x^2(h_2)_1 \beta S((h_2)_2)\cdot\\
&S(X^2 x^3)\alpha X^3)\cdot b\\
=&\sum (X_1^1 x^1 h\cdot a)(X_2^1 x^2 \beta S(X^2 x^3)\alpha X^3)\cdot b\\
=&\sum (x^1 X^1 h\cdot a)(x^2 Y^1 X_1^2 \beta S(x_1^3 Y^2 X_2^2)\cdot\\
&\alpha x_2^3 Y^3 X^3)\cdot b\\
=&\sum (x^1 X^1 h\cdot a)(x^2 Y^1 X_1^2 \beta S(X_2^2)\cdot\\
&S(Y^2)S(x_1^3)\alpha x_2^3 Y^3 X^3)\cdot b\\
=&\sum (x^1 X^1 h\cdot a)(x^2 Y^1 \varepsilon(X^2)\beta S(Y^2)\cdot\\
&\varepsilon(x^3)aY^3 X^3)\cdot b\\
=&\sum (h\cdot a)(Y^1 \beta S(Y^2)aY^3)\cdot b\\
=&(h\cdot a)b.
\end{aligned}
$$

(2)
$$
\begin{aligned}
&\sum x^3 h_2\cdot((S^{-1}(S(x^1)\alpha x^2 h_1 X^1 \beta S(X^2))\cdot\\
&a)(X^3\cdot b))\\
=&\sum x^3\cdot\left[(h_2)_1(S^{-1}(S(x^1)\alpha x^2 h_1 X^1 \beta S\right.\\
&\left.(X^2))\cdot a)((h_2)_2 X^3\cdot b)\right]\\
=&\sum (x_1^3(h_2)_1(S^{-1}(S(x^1)\alpha x^2 h_1 X^1 \beta S\\
&(X^2))\cdot a)(x_2^3(h_2)_2 X^3\cdot b)\\
=&\sum (x_1^3 S^{-1}(S(x^1)\alpha x^2 h_1 X^1 \beta S(X^2)S\\
&(h_2)_1)\cdot a)(x_2^3(h_2)_2 X^3\cdot b)\\
=&\sum (x_1^3 S^{-1}(S(x^1)\alpha x^2 h_1 X^1 \beta S((h_2)_1 X^2))\cdot\\
&a)(x_2^3(h_2)_2 X^3\cdot b)\\
=&\sum (x_1^3 S^{-1}(S(x^1)\alpha x^2 X^1 (h_1)_1 \beta S(X^2\\
&(h_1)_2))\cdot a)(x_2^3 X^3 h_2\cdot b)\\
=&\sum (S^{-1}(S(x^1)\alpha x^2 X^1 (h_1)_1 \beta S(x_1^3 X^2\\
&(h_1)_2))\cdot a)(x_2^3 X^3 h_2\cdot b)\\
=&\sum (S^{-1}(S(x^1)\alpha x^2 X^1 (h_1)_1 \beta S((h_1)_2)S\\
&(x_1^3 X^2))\cdot a)(x_2^3 X^3 h_2\cdot b)\\
=&\sum (S^{-1}(S(x^1)\alpha x^2 X^1 \beta S(x_1^3 X^2))\cdot a)\\
&(x_2^3 X^3 h\cdot b)\\
=&\sum (S^{-1}(S(X_1^1 y^1 x^1)\alpha X_2^1 y^2 x_1^2 \beta S(X^2 y^3 x_2^2))\cdot\\
&a)(X^3 x^3 h\cdot b)\\
=&\sum (S^{-1}(S(x^1)S(y^1)S(X_1^1)\alpha X_2^1 y^2 x_1^2 \beta S\\
&(x_2^2)S(y^3)S(X^2))\cdot a)(X^3 x^3 h\cdot b)\\
=&\sum (S^{-1}(S(x^1)S(y^1)\varepsilon(X^1)\alpha y^2 \varepsilon(x^2)\beta S\\
&(y^3)S(X^2))\cdot a)(X^3 x^3 h\cdot b)\\
=&\sum (S^{-1}(S(y^1)\alpha y^2 \beta S(y^3))\cdot a)(h\cdot b)\\
=&a(h\cdot b).
\end{aligned}
$$

§3 Maschke 型定理

定义 3.1 令 H 是拟 Hopf 代数,元素 $t\in H$ 称做 H 的左(右)积分,若 $ht=\varepsilon(h)t$($th=t\varepsilon(h)$),$\forall h\in H$,H 的全体左(右)积分记作$\int_l$($\int_r$).

$\int_l(\int_r)$ 是 H 的双边理想.

命题 3.1 设 $H=(H,\Delta,\varepsilon,S,\varphi,\alpha,\beta)$ 是有限维拟 Hopf 代数,k - 向量空间 V,W 是左 H - 模,存在右积分 $t\in\int_r$,若 $\lambda: V\to W$ 是 k - 线性的,则 $\tilde{\lambda}: V\to W$,$v\to\sum S(t_1 x^1)\cdot\lambda(\alpha t_2 x^2\beta S(x^3)\cdot v)$ 是 H - 模同态,这里,$\varphi=\sum X^1\otimes X^2\otimes X^3$,$\varphi^{-1}=\sum x^1\otimes x^2\otimes x^3$,$\Delta t=\sum t_1\otimes t_2$.

证明 若 $g\in H$,$v\in V$,要证明:$\tilde{\lambda}(g\cdot v)=g\cdot\tilde{\lambda}(v)$,由于 S 是到上的,故 $g=S(h)$,则

$$
\begin{aligned}
&g\cdot\tilde{\lambda}(v)\\
=&S(h)\sum S(t_1 x^1)\cdot\lambda(\alpha t_2 x^2\beta S(x^3)\cdot v)\\
=&\sum S(h)S(t_1 x^1)\cdot\lambda(\alpha t_2 x^2\beta S(x^3)\cdot v)\\
=&\sum S(h_1)S(t_1 x^1)\cdot\lambda(\alpha t_2 x^2\varepsilon(h_2)\beta S(x^3)\cdot v)\\
=&\sum S(t_1 x^1 h_1)\cdot\lambda(\alpha t_2 x^2 h_{21}\beta S(h_{22})S(x^3)\cdot v)\\
=&\sum S(t_1 x^1 h_1)\cdot\lambda(\alpha t_2 x^2 h_{21}\beta S(x^3 h_{22})\cdot v)\\
=&\sum S(t_1 x^1 X^1 h_{11} y^1)\cdot\lambda(\alpha t_2 x^2 X^2 h_{12} y^2\beta S(x^3 X^3 h_2 y^3)\cdot v)\\
=&\sum S(t_1 h_{11} y^1)\cdot\lambda(\alpha t_2 h_{12} y^2\beta S(h_2 y^3)\cdot v)\\
=&\sum S(t_1 y^1)\cdot\lambda(\alpha t_2\varepsilon(h_1)y^2\beta S(h_2 y^3)\cdot v)\quad(\text{注})\\
=&\sum S(t_1 y^1)\cdot\lambda(\alpha t_2 y^2\beta S(hy^3)\cdot v)\\
=&\sum S(t_1 y^1)\cdot\lambda(\alpha t_2 y^2\beta S(y^3)S(h)\cdot v)\\
=&\tilde{\lambda}(S(h)\cdot v)\\
=&\tilde{\lambda}(g\cdot v)
\end{aligned}
$$

注: $th=t\varepsilon(h)\Rightarrow\sum t_1 h_1\otimes t_2 h_2=\sum t_1\otimes t_2\varepsilon(h)$

定理 3.2 设 $H=(H,\Delta,\varepsilon,S,\varphi,\alpha,\beta)$ 是有限维拟 Hopf 代数,k - 向量空间 V,W 是左 H - 模,$t\in\int_r$,满足 $\varepsilon(t)=1$,若 W 是 V 的 k - 直和项,则 W 是 V

(C)1994-2020 China Academic Journal Electronic Publishing House. All rights reserved. http://www.cnki.net

的 H – 直和项.

证明　令 $\lambda: V \to W$ 是 k – 投射映射, $t \in \int_r$, 满足 $\varepsilon(t) = 1$, 设 $\Delta t = \sum t_1 \otimes t_2$, 定义 $\tilde{\lambda}: V \to W$, $v \to \sum S(t_1 x^1) \cdot \lambda(\alpha t_2 x^2 \beta S(x^3) \cdot v)$, 则 $\tilde{\lambda}$ 是 H – 模同态, 这里, $\varphi = \sum X^1 \otimes X^2 \otimes X^3$, $\varphi^{-1} = \sum x^1 \otimes x^2 \otimes x^3$, 下面只需证明 $\tilde{\lambda}$ 是投射即可

设 $w \in W$, 则 $\alpha t_2 x^2 \beta S(x^3) \cdot w \in W$, 因此, $\lambda(\alpha t_2 x^2 \beta S(x^3) \cdot w) = \alpha t_2 x^2 \beta S(x^3) \cdot w$, 则

$$\begin{aligned}\tilde{\lambda}(w) &= \sum S(t_1 x^1) \cdot \lambda(\alpha t_2 x^2 \beta S(x^3) \cdot w)\\ &= \sum S(t_1 x^1) \alpha t_2 x^2 \beta S(x^3) \cdot w\\ &= \sum S(x^1) S(t_1) \alpha t_2 x^2 \beta S(x^3) \cdot w\\ &= \sum \varepsilon(t) S(x^1) \alpha x^2 \beta S(x^3) \cdot w\\ &= \sum S(x^1) \alpha x^2 \beta S(x^3) \cdot w\\ &= w\end{aligned}$$

定理3.3　设 $H = (H, \Delta, \varepsilon, S, \varphi, \alpha, \beta)$ 是有限维拟 Hopf 代数, k – 向量空间 V, W 是左 H – 模, $t \in \int_r$, 满足 $\varepsilon(t) = 1$, 若 P 的 k – 投射的, 则 P 的 H – 投射的.

证明　设 $f: V \to W$ 是任意 H – 满同态, $g: P \to N$ 是任意 H – 同态, 由于 P 的 k – 投射的, 从而存在 k – 同态 $\lambda: P \to M$, 使得 $f\lambda = g$, 定义 $\tilde{\lambda}: P \to M$, $v \to \sum S(t_1 x^1) \cdot \lambda(\alpha t_2 x^2 \beta S(x^3) \cdot v)$, 则 $\tilde{\lambda}$ 是 H – 模同态, 这里 $t \in \int_r$, 满足 $\varepsilon(t) = 1$, $\Delta t = \sum t_1 \otimes t_2$, $\varphi = \sum X^1 \otimes X^2 \otimes X^3$, $\varphi^{-1} = \sum x^1 \otimes x^2 \otimes x^3$, 使得

$$\begin{aligned}f\tilde{\lambda}(v) &= \sum fS(t_1 x^1) \cdot \lambda(\alpha t_2 x^2 \beta S(x^3) \cdot v)\\ &= \sum S(t_1 x^1) \cdot f\lambda(\alpha t_2 x^2 \beta S(x^3) \cdot v)\\ &= \sum S(t_1 x^1) g(\alpha t_2 x^2 \beta S(x^3) \cdot v)\\ &= \sum S(t_1 x^1) \alpha t_2 x^2 \beta S(x^3) \cdot g(v)\\ &= \sum S(x^1) S(t_1) \alpha t_2 x^2 \beta S(x^3) \cdot g(v)\\ &= \sum \varepsilon(t) S(x^1) \alpha x^2 \beta S(x^3) \cdot g(v)\\ &= \sum S(x^1) \alpha x^2 \beta S(x^3) \cdot g(v)\\ &= g(v)\end{aligned}$$

所以, $f\tilde{\lambda} = g$.

参 考 文 献

[1] D. Bulacu F. Panaite and F. Van Oystaeyen, Quasi-Hopf algebra actions and smash products [J]. Commun. Algebra, 2000, 28(2): 631 – 651.

[2] D. Bulacu and E. Nauwelaerts, Relative Hopf Modules for (Dual) Quasi-Hopf Algebras [J]. J. of algebra 2000, 229: 632 – 659.

[3] C. Kassel, "Quantum Groups" [M]. Springer-Verlag, New York/Berlin, 1995.

[4] D. Bulacu and S. Caenepeel, The Quantum Double for Quasitriangular Quasi-Hopf algebra [J]. Commun. Algebra, 2003, 31(3): 1403 – 1425.

[5] F. W. Anderson & K. P. Fuller, Rings and categories of modules [M]. Springer-Verlag, New York, 1974.

[6] V. G. Drinfel'd, Quantum groups, In "Proc. International Congress on Math., Berkeley, 1986," p. 798.

[7] T. Albu, Năstăsescu C. Relative Finiteness in ModuleTheory [M]. New York: Dekker, 1984.

[8] H. Bass On the ubiquity of Gorenstein rings [J], Math. Z., 1963, 82: 8 – 28.

[9] Y. Doi Hopf extension of algebras and Masche-type theorems [J]. Israel J. Math., 1990, 72: 99 – 108.

[10] E E. Enochs, Garcia J J., del Rio A. When does R Gorenstein imply R^G Gorenstein? [J]. J. Algebra, 1996, 182: 561 – 576.

[11] H. Matsumura Commutative Ring Theory [M]. Cambridge: Cambridge University Press, 1986.

[12] S. Montgomery Hopf Algebra and Their Actions on Rings [D]. CBMS Regional Conference Series in Math. Number 82, AMS, Providence, 1993.

[13] S. Montgomery Hopf Galois extension [J]. AMS Contemporary Math., 1992, 124: 129 – 140.

[14] J J. Rotman An Introduction to Homological Algebra [M]. New York: Academic Press, 1979.

[15] B. Stenström Rings of Quotients [M]. Berlin: Springer, 1975.

[16] F. Van Oystaeyen, Xu Y H., Zang Y H. Inductions and coinductions for Hopf extension [J]. Science in China (Series

(C)1994-2020 China Academic Journal Electronic Publishing House. All rights reserved.　http://www.cnki.net

A),1996,39: 246 – 263.

[17] S L. Yang, J N. Chen Hopf actions on factor rings and induced Hopf actions [J]. Comm. in Algebra,1997, 25: 1103 – 1110.

[18] F. Panaite A Maschke-type Theorem for Quasi-Hopf Algebras in "In Rings Hopf algebeas and Brauer groups" [M]. Dekker,New York,1998: 201 – 207.

[19] D. Bulacu ,S. Caenepeel. Intergrals for (dual) quasi-Hopf algebras. Applications, QA/0110063v2 18 Oct 2002.

Maschke Type Theorem for Quasi-Hopf Algebra

Yang Cunjie

(School of Mathematical Sciences, Capital Normal University,Beijing 100048)

Abstract

We prove that Maschke type theorem for quasi-Hopf algebra.

Key words: quasi-Hopf algebra, gauge transformation, quasi two-cocycle, Maschke type theorem.

(上接第 5 页)

参 考 文 献

[1] Underwood Dudley. Elementary Number Theory (Second Edition). W. H. Freeman and Company, San Francisco,1978.

[2] Martin Aigner, Günter M. Ziegler. Proofs from THE BOOK(中译本) 数学天书中的证明(冯荣权,宋春伟,宗传明译). 北京: 高等教育出版社,2009.

[3] 柯召,孙琦. 数论讲义. 北京: 高等教育出版社,1986.

On the Uniqueness for an Integer to be a Sum of Two Squares

Zhu Yixin

(School of Mathematical Sciences, Captial Normal University, Beijing 100048)

Abstract

For an integer n which is a sum of two integer squares, such that $n = a^2 + b^2$, we give the formula to counting the number of pairs (a, b), by prime factors of n. And then we get a necessary and sufficient condition for an integer to be a sum of two integer squares in uniquness way.

Key words: sum of two integer squares, prime factor, unique factorization domain.

(C)1994-2020 China Academic Journal Electronic Publishing House. All rights reserved. http://www.cnki.net