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广义 Fibonacci 序列与若干变换*

张福玲

(渭南师范学院数学与信息科学学院,陕西 渭南 714000)

摘 要

本文建立了一类广义 Fibonacci 序列与 Secant 变换、Newton-Raphson 变换和 Halley 变换的巧妙联系,对现有的结果进行了推广.

关键词: 广义 Fibonacci 序列,比率,变换.

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1 引 言

著名的 Fibonacci $\{F_n\}$ 数列是由二次线性递推公式 $F_{n+2} = F_{n+1} + F_n n \geqslant 0$ 所定义,其中 $F_0 = 1, F_1 = 1$. 在参考文献 [1] 中 Phillips 首先提出了 Fibonacci 序列与 Aitken 变换之间的关系:

$$A(r_{n-t}, r_n, r_{n+t}) = r_{2n},$$

其中 $r_n = F_{n+1}/F_n$;在参考文献 [2-5] 中 J. H. McCabe 和 G. M. Phillis [2], M. J. Jamieson [3], J. B. Muskat [4], Zhang Zhizheng [5] 相继推广了这个工作,讨论了通常广义的 Fibonacci 序列

$$u_0 = 0, u_1 = 1, u_{n+2} = pu_{n+1} - qu_n, n \geqslant 1,$$

和 Lucas 序列

$$v_0 = 0, v_1 = 1, v_{n+2} = pv_{n+1} - qv_n, n \geqslant 1,$$

与 Aitken 变换、Secant 变换,Newton-Raphson 变换和 Halley 变换的巧妙联系,得到了

$$A(r_{n-t}, r_n, r_{n+t}) = r_{2n}, A(R_{n-t}, R_n, R_{n+t}) = r_{2n};$$
$$S(r_n, r_m) = r_{n+m}, S(R_n, R_m) = r_{n+m};$$
$$N(r_n) = r_{2n}, N(R_n) = r_{2n};$$
$$H(r_n) = r_{3n}, H(R_n) = r_{3n};$$

这里 $r_n = u_{n+d}/u_n, R_n = v_{n+d}/v_n$.

在文献 [5] 中设方程 $\lambda^2 - p\lambda + q = 0$ 的两根为:

$$\alpha = \frac{1}{2}(p + \sqrt{p^2 - 4q}), \beta = \frac{1}{2}(p - \sqrt{p^2 - 4q}),$$

定义广义的 Fibonacci 序列定义为:

$$W_{n,d}^{(k)}(A, B) = \frac{A^k\alpha^{nk+d} - B^k\beta^{nk+d}}{\alpha - \beta}, \quad (1)$$

$W_{n,d}^{(k)}(A, B)$ 满足下列递归关系

$$W_{n,d}^{(k)}(A, B) = (\alpha^k + \beta^k)W_{n-1,d}^{(k)}(A, B) - (\alpha^k\beta^k)W_{n-2,d}^{(k)}(A, B), (n \geqslant 2), W_{0,d}^{(k)}(A, B) = a,$$
$$W_{1,d}^{(k)}(A, B) = b,$$

则通常的广义 Fibonacci 序列和 Lucas 序列为:

$$U_n(p, q) = W_{n,0}^{(1)}(1, 1),$$
$$V_n(p, q) = W_{n,0}^{(1)}(\sqrt{p^2 - 4q}, -\sqrt{p^2 - 4q}),$$

普通的 Fibonacci 序列和 Lucas 序列为:

$$F_n = U_n(1, -1), L_n = V_n(1, -1),$$

并讨论了更一般的 Fibonacci 序列 $\{W_{n,d}^{(k)}\}$ 与 Aitken 变换的联系,得到了:

$$A(r_{n-t}^{(k)}(A_1, A_2; B_1, B_2; d_1, d_2), r_n^{(k)}(A_1, A_2; B_1, B_2; d_1, d_2), r_{n+t}^{(k)}(A_1, A_2; B_1, B_2; d_1, d_2)) = r_{2n}^{(k)}(A_1A_2, B_1B_2; A_2^2, B_2^2; d_1 + d_2, 2d_2),$$

这里 $r_n^{(k)}(A_1, A_2; B_1, B_2; d_1, d_2) = \frac{W_{n,d_1}^{(k)}(A_1, B_1)}{W_{n,d_2}^{(k)}(A_2, B_2)}$.

在参考文献 [7] 中分别定义了 Secant 变换,Newton-Raphson 变换和 Halley 变换:

$$S(x, x') = \frac{xf(x') - x'f(x)}{f(x') - f(x)}.$$

$$N(x) = x - \frac{f(x)}{f'(x)}.$$

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$$H(x) = x - \frac{f(x)}{f'(x) - \frac{f(x)f''(x)}{2f'(x)}}.$$

本文通过定义(1),将参考文献[2-5]中的结果继续推广,建立更一般的 Fibonacci 序列 $\{W_{n,d}^{(k)}\}$ 与 Secant 变换,Newton-Raphson 变换和 Halley 变换的联系.

2 主要引理

引理 1

$(A_2B_2)^k W_{n,d_1}^{(k)}(A_1,B_1)W_{m,d_1}^{(k)}(A_1,B_1) - (A_1B)^k(\alpha\beta)^{d_1-d_2}W_{n,d_2}^{(k)}(A_2,B_2)W_{m,d_2}^{(k)}(A_2,B_2) = W_{m+n,d_1+d_2}^{(k)}(A_1A_2,B_1B_2)W_{0,d_1-d_2}^{(k)}(A_1B_2,A_2B_1)$.

证明: 利用 $W_{n,d}^{(k)}(A,B)$ 的定义可知

$$\begin{aligned}
&(A_2B_2)^k W_{n,d_1}^{(k)}(A_1,B_1)W_{m,d_1}^{(k)}(A_1,B_1) - (A_1B_1)^k(\alpha\beta)^{d_1-d_2}W_{n,d_2}^{(k)}(A_2,B_2)W_{m,d_2}^{(k)}(A_2,B_2)\\
&= (A_2B_2)^k\frac{A_1^k\alpha^{nk+d_1}-B_1^k\beta^{nk+d_1}}{\alpha-\beta}\frac{A_1^k\alpha^{mk+d_1}-B_1^k\beta^{mk+d_1}}{\alpha-\beta} - (A_1B_1)^k(\alpha\beta)^{d_1-d_2}\frac{A_2^k\alpha^{nk+d_2}-B_2^k\beta^{nk+d_2}}{\alpha-\beta}\frac{A_2^k\alpha^{mk+d_2}-B_2^k\beta^{mk+d_2}}{\alpha-\beta}\\
&= \frac{(A_1^k\alpha^{nk+d_1}-B_1^k\beta^{nk+d_1})(A_1^k\alpha^{mk+d_1}-B_1^k\beta^{mk+d_1})(A_2B_2)^k - (A_2^k\alpha^{nk+d_2}-B_2^k\beta^{nk+d_2})(A_2^k\alpha^{mk+d_2}-B_2^k\beta^{mk+d_2})(A_1B_1)^k(\alpha\beta)^{d_1-d_2}}{(\alpha-\beta)^2}\\
&= \frac{\alpha^{d_1-d_2}(A_1B_2)^k((A_1A_2)^k\alpha^{(m+n)k+(d_1+d_2)}-(B_1B_2)^k\beta^{(m+n)k+(d_1+d_2)}) - \beta^{d_1-d_2}(A_2B_1)^k((A_1A_2)^k\alpha^{(m+n)k+(d_1+d_2)}-(B_1B_2)^k\beta^{(m+n)k+(d_1+d_2)})}{(\alpha-\beta)^2}\\
&= \frac{((A_1A_2)^k\alpha^{(m+n)k+(d_1+d_2)}-(B_1B_2)^k\beta^{(m+n)k+(d_1+d_2)})(\alpha^{d_1-d_2}(A_1B_2)^k-\beta^{d_1-d_2}(A_2B_1)^k)}{(\alpha-\beta)^2}\\
&= \frac{((A_1A_2)^k\alpha^{(m+n)k+(d_1+d_2)}-(B_1B_2)^k\beta^{(m+n)k+(d_1+d_2)})}{(\alpha-\beta)}\cdot\frac{((A_1B_2)^k\alpha^{d_1-d_2}-(A_2B_1)^k\beta^{d_1-d_2})}{(\alpha-\beta)}\\
&= W_{m+n,d_1+d_2}^{(k)}(A_1A_2,B_1B_2)W_{0,d_1-d_2}^{(k)}(A_1B_2,A_2B_1).
\end{aligned}$$

同理可证以下引理:

引理 2

$(A_2B_2)^k(W_{n,d_1}^{(k)}(A_1,B_1)W_{m,d_2}^{(k)}(A_2,B_2) + W_{m,d_1}^{(k)}(A_1,B_1)W_{n,d_2}^{(k)}(A_2,B_2)) - ((A_1B_2)^k\alpha^{d_1-d_2} + (A_2B_1)^k\beta^{d_1-d_2})W_{n,d_2}^{(k)}(A_2,B_2)W_{m,d_2}^{(k)}(A_2,B_2) = W_{m+n,2d_2}^{(k)}(A_2^2,B_2^2)W_{0,d_1-d_2}^{(k)}(A_1B_2,A_2B_1)$.

引理 3

$(A_2B_2)^k(W_{n,d_1}^{(k)}(A_1,B_1))^2 - (A_1B_1)^k(\alpha\beta)^{d_1-d_2}(W_{n,d_2}^{(k)}(A_2,B_2))^2 = W_{2n,d_1+d_2}^{(k)}(A_1A_2,B_1B_2)\ W_{0,d_1-d_2}^{(k)}(A_1B_2,A_2B_1)$.

引理 4

$2(A_2B_2)^k W_{n,d_1}^{(k)}(A_1,B_1)W_{n,d_2}^{(k)}(A_2,B_2) - ((A_1B_2)^k\alpha^{d_1-d_2} + (A_2B_1)^k\beta^{d_1-d_2})(W_{n,d_2}^{(k)}(A_2,B_2))^2 = W_{2n,2d_2}^{(k)}(A_2^2,B_2^2)W_{0,d_1-d_2}^{(k)}(A_1B_2,A_2B_1)$.

引理 5

$(A_2^2B_2^2)^k(W_{n,d_1}^{(k)}(A_1,B_1))^3 - 3(A_1B_1A_2B_2)^k(\alpha\beta)^{d_1-d_2}W_{n,d_1}^{(k)}(A_1,B_1)(W_{n,d_2}^{(k)}(A_2,B_2))^2 + ((A_1B_2)^k\alpha^{d_1-d_2} + (A_2B_1)^k\beta^{d_1-d_2})(A_1B_1)^k(\alpha\beta)^{d_1-d_2}(W_{n,d_2}^{(k)}(A_2,B_2))^3 = W_{3n,d_1|+2d_2}^{(k)}(A_1A_2^2,B_1B_2^2)(W_{0,d_1-d_2}^{(k)}(A_1B_2,A_2B_1))^2$.

引理 6

$3(W_{n,d_1}^{(k)}(A_1,B_1))^2W_{n,d_2}^{(k)}(A_2,B_2)(A_2^2B_2^2)^k - 3((A_1B_2)^k\alpha^{d_1-d_2} + (A_2B_1)^k\beta^{d_1-d_2})(A_2B_2)^kW_{n,d_1}^{(k)}(A_1,B_1)(W_{n,d_2}^{(k)}(A_2,B_2))^2 + ((A_1B_2)^k\alpha^{d_1-d_2} + (A_2B_1)^k\beta^{d_1-d_2})^2(W_{n,d_2}^{(k)}(A_2,B_2))^3 - (A_1B_1A_2B_2)^k(\alpha\beta)^{d_1-d_2}(W_{n,d_2}^{(k)}(A_2,B_2))^3 = W_{3n,,3d_2}^{(k)}(A_2^3,B_2^3)(W_{0,d_1-d_2}^{(k)}(A_1B_2,A_2B_1))^2$.

3 主要结果及证明

序列 $\{r_n^{(k)}\}$ 收敛于方程

$$x^2 - \left[\left(\frac{A_1}{A_2}\right)^k\alpha^{d_1-d_2} + \left(\frac{B_1}{B_2}\right)^k\beta^{d_1-d_2}\right]x + \left(\frac{A_1B_1}{A_2B_2}\right)^k(\alpha\beta)^{d_1-d_2} = 0$$

的一个根[4],对于此方程 Secant、Newton-Raphson **和** Halley **变换分别为:**

$$S(x,x') = \frac{xx' - \left(\frac{A_1B_1}{A_2B_2}\right)^k(\alpha\beta)^{d_1-d_2}}{x'+x - \left(\left(\frac{A_1}{A_2}\right)^k\alpha^{d_1-d_2} + \left(\frac{B_1}{B_2}\right)^k\beta^{d_1-d_2}\right)}.$$

$$N(x) = \frac{x^2 - \left(\frac{A_1B_1}{A_2B_2}\right)^k(\alpha\beta)^{d_1-d_2}}{2x - \left(\left(\frac{A_1}{A_2}\right)^k\alpha^{d_1-d_2} + \left(\frac{B_1}{B_2}\right)^k\beta^{d_1-d_2}\right)}.$$

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$$H(x)=\frac{x^3-3\left(\frac{A_1B_1}{A_2B_2}\right)^k(\alpha\beta)^{d_1-d_2}x+\left(\left(\frac{A_1}{A_2}\right)^k\alpha^{d_1-d_2}+\left(\frac{B_1}{B_2}\right)^k\beta^{d_1-d_2}\right)\left(\frac{A_1B_1}{A_2B_2}\right)^k(\alpha\beta)^{d_1-d_2}}{3x^2-3\left(\left(\frac{A_1}{A_2}\right)^k\alpha^{d_1-d_2}+\left(\frac{B_1}{B_2}\right)^k\beta^{d_1-d_2}\right)x+\left(\left(\frac{A_1}{A_2}\right)^k\alpha^{d_1-d_2}+\left(\frac{B_1}{B_2}\right)^k\beta^{d_1-d_2}\right)^2-\left(\frac{A_1B_1}{A_2B_2}\right)^k(\alpha\beta)^{d_1-d_2}}.$$

定理 1 $S(r_n^{(k)},r_m^{(k)})=r_{m+n}^{(k)}(A_1A_2,B_1B_2;A_2^2,B_2^2;d_1+d_2,2d_2)$. **证明:**

$$\begin{aligned}
&S(r_n^{(k)},r_m^{(k)})\\
&=\frac{r_n^{(k)}r_m^{(k)}-\left(\frac{A_1B_1}{A_2B_2}\right)^k(\alpha\beta)^{d_1-d_2}}{r_n^{(k)}+r_m^{(k)}-\left(\left(\frac{A_1}{A_2}\right)^k\alpha^{d_1-d_2}+\left(\frac{B_1}{B_2}\right)^k\beta^{d_1-d_2}\right)}\\
&=\frac{\frac{W_{n,d_1}^{(k)}(A_1,B_1)}{W_{n,d_2}^{(k)}(A_2,B_2)}\frac{W_{m,d_1}^{(k)}(A_1,B_1)}{W_{m,d_2}^{(k)}(A_2,B_2)}-\left(\frac{A_1B_1}{A_2B_2}\right)^k(\alpha\beta)^{d_1-d_2}}{\frac{W_{n,d_1}^{(k)}(A_1,B_1)}{W_{n,d_2}^{(k)}(A_2,B_2)}+\frac{W_{m,d_1}^{(k)}(A_1,B_1)}{W_{m,d_2}^{(k)}(A_2,B_2)}-\left(\left(\frac{A_1}{A_2}\right)^k\alpha^{d_1-d_2}+\left(\frac{B_1}{B_2}\right)^k\beta^{d_1-d_2}\right)}\\
&=\frac{W_{n,d_1}^{(k)}(A_1,B_1)W_{m,d_1}^{(k)}(A_1,B_1)-\left(\frac{A_1B_1}{A_2B_2}\right)^k(\alpha\beta)^{d_1-d_2}W_{n,d_2}^{(k)}(A_2,B_2)W_{m,d_2}^{(k)}(A_2,B_2)}{W_{n,d_1}^{(k)}(A_1,B_1)W_{m,d_2}^{(k)}(A_2,B_2)+W_{m,d_1}^{(k)}(A_1,B_1)W_{n,d_2}^{(k)}(A_2,B_2)-\left(\left(\frac{A_1}{A_2}\right)^k\alpha^{d_1-d_2}+\left(\frac{B_1}{B_2}\right)^k\beta^{d_1-d_2}\right)W_{n,d_2}^{(k)}(A_2,B_2)W_{m,d_2}^{(k)}(A_2,B_2)}\\
&=\frac{(A_2B_2)^kW_{n,d_1}^{(k)}(A_1,B_1)W_{m,d_1}^{(k)}(A_1,B_1)-(A_1B_1)^k(\alpha\beta)^{d_1-d_2}W_{n,d_2}^{(k)}(A_2,B_2)W_{m,d_2}^{(k)}(A_2,B_2)}{(A_2B_2)^k(W_{n,d_1}^{(k)}(A_1,B_1)W_{m,d_2}^{(k)}(A_2,B_2)+W_{m,d_1}^{(k)}(A_1,B_1)W_{n,d_2}^{(k)}(A_2,B_2))-((A_1B_2)^k\alpha^{d_1-d_2}+(A_2B_1)^k\beta^{d_1-d_2})W_{n,d_2}^{(k)}(A_2,B_2)W_{m,d_2}^{(k)}(A_2,B_2)}\\
&=\frac{W_{m+n,d_1+d_2}^{(k)}(A_1A_2,B_1B_2)W_{0,d_1-d_2}^{(k)}(A_1B_2,A_2B_1)}{W_{m+n,2d_2}^{(k)}(A_2^2,B_2^2)W_{0,d_1-d_2}^{(k)}(A_1B_2,A_2B_1)}\text{(应用引理 1、引理 2)}\\
&=r_{m+n}^{(k)}(A_1A_2,B_1B_2;A_2^2,B_2^2;d_1+d_2,2d_2).
\end{aligned}$$

定理 2 $N(r_n^{(k)})=r_{2n}^{(k)}(A_1A_2,B_1B_2;A_2^2,B_2^2;d_1+d_2,2d_2)$. **证明:**

$$\begin{aligned}
N(r_n^{(k)})&=\frac{(r_n^{(k)})^2-\left(\frac{A_1B_1}{A_2B_2}\right)^k(\alpha\beta)^{d_1-d_2}}{2r_n^{(k)}-\left(\left(\frac{A_1}{A_2}\right)^k\alpha^{d_1-d_2}+\left(\frac{B_1}{B_2}\right)^k\beta^{d_1-d_2}\right)}\\
&=\frac{\left(\frac{W_{n,d_1}^{(k)}(A_1,B_1)}{W_{n,d_2}^{(k)}(A_2,B_2)}\right)^2-\left(\frac{A_1B_1}{A_2B_2}\right)^k(\alpha\beta)^{d_1-d_2}}{2\frac{W_{n,d_1}^{(k)}(A_1,B_1)}{W_{n,d_2}^{(k)}(A_2,B_2)}-\left(\left(\frac{A_1}{A_2}\right)^k\alpha^{d_1-d_2}+\left(\frac{B_1}{B_2}\right)^k\beta^{d_1-d_2}\right)}\\
&=\frac{(W_{n,d_1}^{(k)}(A_1,B_1))^2-\left(\frac{A_1B_1}{A_2B_2}\right)^k(\alpha\beta)^{d_1-d_2}(W_{n,d_2}^{(k)}(A_2,B_2))^2}{2W_{n,d_1}^{(k)}(A_1,B_1)W_{n,d_2}^{(k)}(A_2,B_2)-\left(\left(\frac{A_1}{A_2}\right)^k\alpha^{d_1-d_2}+\left(\frac{B_1}{B_2}\right)^k\beta^{d_1-d_2}\right)(W_{n,d_2}^{(k)}(A_2,B_2))^2}\\
&=\frac{(A_2B_2)^k(W_{n,d_1}^{(k)}(A_1,B_1))^2-(A_1B_1)^k(\alpha\beta)^{d_1-d_2}(W_{n,d_2}^{(k)}(A_2,B_2))^2}{2(A_2B_2)^kW_{n,d_1}^{(k)}(A_1,B_1)W_{n,d_2}^{(k)}(A_2,B_2)-((A_1B_2)^k\alpha^{d_1-d_2}+(A_2B_1)^k\beta^{d_1-d_2})(W_{n,d_2}^{(k)}(A_2,B_2))^2}\\
&=\frac{W_{2n,d_1+d_2}^{(k)}(A_1A_2,B_1B_2)W_{0,d_1-d_2}^{(k)}(A_1B_2,A_2B_1)}{W_{2n,2d_2}^{(k)}(A_2^2,B_2^2)W_{0,d_1-d_2}^{(k)}(A_1B_2,A_2B_1)}\text{(应用引理 3、引理 4)}\\
&=r_{2n}^{(k)}(A_1A_2,B_1B_2;A_2^2,B_2^2;d_1+d_2,2d_2).
\end{aligned}$$

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定理 3　$H(r_n^{(k)}) = r_{3n}^{(k)}(A_1A_2^2, B_1B_2^2; A_2^3, B_2^3; d_1 + 2d_2, 3d_2)$.　　**证明:**

$$H(r_n^{(k)}) = \frac{(r_n^{(k)})^3 - 3\left(\frac{A_1B_1}{A_2B_2}\right)^k(\alpha\beta)^{d_1-d_2}r_n^{(k)} + \left(\left(\frac{A_1}{A_2}\right)^k\alpha^{d_1-d_2} + \left(\frac{B_1}{B_2}\right)^k\beta^{d_1-d_2}\right)\left(\frac{A_1B_1}{A_2B_2}\right)^k(\alpha\beta)^{d_1-d_2}}{3(r_n^{(k)})^2 - 3\left(\frac{A_1}{A_2}\right)^k\alpha^{d_1-d_2} + \left(\frac{B_1}{B_2}\right)^k\beta^{d_1-d_2})r_n^{(k)} + \left(\left(\frac{A_1}{A_2}\right)^k\alpha^{d_1-d_2} + \left(\frac{B_1}{B_2}\right)^k\beta^{d_1-d_2}\right)^2 - \left(\frac{A_1B_1}{A_2B_2}\right)^k(\alpha\beta)^{d_1-d_2}}$$

$$= \frac{(A_2^2B_2^2)^k(r_n^{(k)})^3 - 3(A_1B_1A_2B_2)^k(\alpha\beta)^{d_1-d_2}r_n^{(k)} + ((A_1B_2)^k\alpha^{d_1-d_2} + (A_2B_1)^k\beta^{d_1-d_2})(A_1B_1)^k(\alpha\beta)^{d_1-d_2}}{3(r_n^{(k)})^2(A_2^2B_2^2)^k - 3((A_1B_2)^k\alpha^{d_1-d_2} + (A_2B_1)^k\beta^{d_1-d_2})(A_2B_2)^kr_n^{(k)} + ((A_1B_2)^k\alpha^{d_1-d_2} + (A_2B_1)^k\beta^{d_1-d_2})^2 - (A_1B_1A_2B_2)^k(\alpha\beta)^{d_1-d_2}}$$

$$= \frac{(A_2^2B_2^2)^k\left(\frac{W_{n,d_1}^{(k)}(A_1,B_1)}{W_{n,d_2}^{(k)}(A_2,B_2)}\right)^3 - 3(A_1B_1A_2B_2)^k(\alpha\beta)^{d_1-d_2}\frac{W_{n,d_1}^{(k)}(A_1,B_1)}{W_{n,d_2}^{(k)}(A_2,B_2)} + ((A_1B_2)^k\alpha^{d_1-d_2} + (A_2B_1)^k\beta^{d_1-d_2})(A_1B_1)^k(\alpha\beta)^{d_1-d_2}}{3\left(\frac{W_{n,d_1}^{(k)}(A_1,B_1)}{W_{n,d_2}^{(k)}(A_2,B_2)}\right)^2(A_2^2B_2^2)^k - 3((A_1B_2)^k\alpha^{d_1-d_2} + (A_2B_1)^k\beta^{d_1-d_2})(A_2B_2)^k\frac{W_{n,d_1}^{(k)}(A_1,B_1)}{W_{n,d_2}^{(k)}(A_2,B_2)} + ((A_1B_2)^k\alpha^{d_1-d_2} + (A_2B_1)^k\beta^{d_1-d_2})^2 - (A_1B_1A_2B_2)^k(\alpha\beta)^{d_1-d_2}}$$

$$= \frac{W_{3n,d_1|+2d_2}^{(k)}(A_1A_2^2, B_1B_2^2)(W_{0,d_1-d_2}^{(k)}(A_1B_2, A_2B_1))}{W_{3n,,3d_2}^{(k)}(A_2^3, B_2^3)(W_{0,d_1-d_2}^{(k)}(A_1B_2, A_2B_1))^2}\text{(应用引理 5、引理 6)}$$

$$= r_{3n}^{(k)}(A_1A_2^2, B_1B_2^2; A_2^3, B_2^3; d_1 + 2d_2, 3d_2).$$

当 $k = 1; A_1 = A_2 = 1; B_1 = B_2 = 1; d_1 = d, d_2 = 0;$ $r_n^{(k)}(A_1, B_1; A_2, B_2; d_1, d_2) = r_n = \frac{U_{n+d}}{U_n}$ **;则**

$$S(r_n, r_m) = \frac{U_{m+n+d}}{U_{m+n}} = r_{m+n}.$$
$$N(r_n) = r_{2n}.$$
$$H(r_n) = r_{3n}.$$

当 $k = 1; A_1 = A_2 = \sqrt{p^2 - 4q}; B_1 = B_2 = -\sqrt{p^2 - 4q}; d_1 = d, d_2 = 0; r_n^{(k)}(A_1, B_1; A_2, B_2; d_1, d_2) = R_n = \frac{V_{n+d}}{V_n}$ **;则**

$$S(R_n, R_m) = \frac{U_{m+n+d}}{U_{m+n}} = r_{m+n}.$$
$$N(R_n) = r_{2n}.$$
$$H(R_n) = R_{3n}.$$

当 $k = 1; A_1 = \sqrt{p^2 - 4q}, A_2 = 1; B_1 = -\sqrt{p^2 - 4q}, B_2 = 1; d_1 = d, d_2 = 0; r_n^{(k)}(A_1, B_1; A_2, B_2; d_1, d_2) = \Omega_n = \frac{V_{n+d}}{U_n}$ **;则**

$$S(\Omega_n, \Omega_m) = \frac{V_{m+n+d}}{U_{m+n}} = \Omega_{m+n}.$$
$$N(\Omega_n) = \Omega_{2n}.$$
$$H(\Omega_n) = \Omega_{3n}.$$

当 $k = 1; A_1 = 1, A_2 = \sqrt{p^2 - 4q}; B_1 = 1, B_2 = -\sqrt{p^2 - 4q}; d_1 = d, d_2 = 0; r_n^{(k)}(A_1, B_1; A_2, B_2; d_1, d_2) = \Psi_n = \frac{U_{n+d}}{V_n}$ **;则**

$$S(\Psi_n, \Psi_m) = \frac{1}{p^2 - 4q}\frac{V_{m+n+d}}{U_{m+n}} = \frac{1}{p^2 - 4q}\Omega_{m+n}.$$
$$N(\Psi_n) = \frac{1}{p^2 - 4q}\Omega_{2n}.$$
$$H(\Psi_n) = \frac{1}{p^2 - 4q}\Omega_{3n}.$$

参 考 文 献

[1] G. M. Phpllips. Aitken sequences and Fibonacci numbers [J]. The American Mathematical Monthly, 1984, 91: 354 – 357.

[2] J. H. McCabe, G. M. Phillips. Aitken sequences and generalized Fibonacci numbers [J]. Mathematics of Computation, 1985, 45:553 – 558.

[3] M. J. Jamieson. Fibonacci numbers and Aitken sequences revisited [J]. The American Mathematical Monthly 1990, 97:829 – 831.

[4] J. B. Muskat. Generalized Fibonacci and Lucas sequences and rootfinding methods [J]. Mathematics of Computation, 1993, 61:365 – 372.

[5] Zhang Zhizheng. A class of sequences and the Aitken transformation [J]. The Fibonacci Quarterly, 1998, 36. (1):68 –

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71.
[6] 张世武,张之正. 广义 Fibonacci 序列与 Aitken 变换的一点注记[J]. 上海大学学报(自然科学版),1999,vol. 5(5):385 - 387.
[7] W. Gander. On Halley. s iteration method [J]. The American Mathematical Monthly,1985, 92: 131 - 134.

Contacts on Generalized Fibonacci Sequence and Some Transformations

Zhang Fuling

(College of Mathematics and Information,Weinan Teacher's College, Weinan Shaanxi 714000)

Abstract

The purpose of this paper is to establish some relations between the generalized Fibonacci and the Secant, Newton-Raphson and Halley transformations. This consequence popularize some previous results.

Key words: generalized fibonacci sequence, ratio, transformation.

(上接第 5 页)

Gröbner-Shirshov Basis of Quantum Group of Type C_3

Sanar. Humax Abdukadir. Obul

(Xinjiang university, Xinjiang Urumqi 830046)

Abstract

In this paper we prove, by using the Auslander-Reiten theory in representation theory of finite dimensional algebra and the Ringel-Hall algebra approach, that the set of all skew-commutator relations between the root vectors of quantized enveloping algebra of type C_3 is a minimal Gröbner-Shirshov basis.

Key words: Ringel-Hall algebra, indecomposable module, skew-commutator relation, Gröbner-Shirshov basis.

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