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I-fuzzy 拓扑线性空间中的凸性*

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摘要: 本文对 Fuzzifying 拓扑线性空间做了推广,给出了 I-fuzzy 拓扑线性空间的定义并重点研究了该空间中的 I-fuzzy 凸性,讨论了 I-fuzzy 凸的相关性质.

关键词: 拓扑线性空间; I-fuzzy 凸; 模糊逻辑; 重域

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0 引 言

随着 Fuzzy 拓扑学的发展,1977 年,Katsaras 和 Liu 在 [2] 中利用 Fuzzy 拓扑引入了 Fuzzy 拓扑线性空间的概念,但是定义中的 Fuzzy 拓扑不具有“平移不变性”,从而无法再继续进行深入地研究. 随后,吴从炘[3],方锦暄[4] 又先后给出 Fuzzy 拓扑线性空间的定义,定义中对线性运算的连续性做了加强,但也存在局限性,为了弥补这种不足,1981 年,Katsaras 对 Fuzzy 拓扑线性空间重新定义[1],这种定义在形式上与拓扑线性空间的定义保持了一致,而且在这样定义的 Fuzzy 拓扑线性空间的理论框架下,研究成果非常丰富,所以这样的定义被人们普遍接受并沿用至今.

2006 年,张广济等[14] 在引入 Fuzzifying 拓扑线性空间的概念,研究了这类空间中零元平衡邻域系的结构及性质,同时还给出 Fuzzifying 凸集的定义并讨论了它的性质. 在以上文章的启发下,本文对 Fuzzifying 拓扑线性空间做了推广,给出了 I-fuzzy 拓扑线性空间的定义并重点研究了该空间中的 I-fuzzy 凸性,讨论了 I-fuzzy 凸的相关性质.

1 预备知识

定义 1.1[11] 设 X 是论域,$\mathcal{T}\in\mathscr{F}(\mathscr{F}(X))$ 满足以下条件:

(1) $\models X\in\mathcal{T}$;

(2) 对于任意的 $\tilde{A}_1,\tilde{A}_2$,$\models(\tilde{A}_1\in\mathcal{T})\wedge(\tilde{A}_2\in\mathcal{T})\to(\tilde{A}_1\cap\tilde{A}_2\in\mathcal{T})$;

(3) 对于任意的 $\{\tilde{A}_\lambda\mid\lambda\in\Lambda\}$,$\models\forall\lambda(\lambda\in\Lambda\to\tilde{A}_\lambda\in\mathcal{T})\to\bigcup\limits_{\lambda\in\Lambda}\tilde{A}_\lambda\in\mathcal{T}$.

则称 $\mathcal{T}$ 为 X 上的 I-fuzzy 拓扑(bifuzzy 拓扑或 smooth 拓扑),称偶对 $(X,\mathcal{T})$ 为 I-fuzzy 拓扑空间(bifuzzy 拓扑空间或 smooth 拓扑空间).

定义 1.2[5] 设 $\mathcal{T}:I^X\to I$ 是 X 上的 I-fuzzy 拓扑,$\forall x_\lambda\in Pt(I^X)$,$\tilde{A}\in I^X$

$$Q_{x_\lambda}(\tilde{A})=\begin{cases}\bigvee\limits_{x_\lambda\hat{q}\tilde{B}\leqslant\tilde{A}}\mathcal{T}(\tilde{B}), & x_\lambda\hat{q}\tilde{A},\\ 0, & x_\lambda\neg\,\hat{q}\tilde{A}.\end{cases}$$

定义 1.3[6] 设 Λ 为任意指标集,$\tilde{A}_\lambda\in\mathscr{F}(X)(\lambda\in\Lambda)$,分别定义并 $\bigcup\limits_{\lambda\in\Lambda}\tilde{A}_\lambda$ 和交 $\bigcap\limits_{\lambda\in\Lambda}\tilde{A}_\lambda$ 如下:

$$(\bigcup_{\lambda\in\Lambda}\tilde{A}_\lambda)(x)=\sup_{\lambda\in\Lambda}\tilde{A}_\lambda(x);\quad(\bigcap_{\lambda\in\Lambda}\tilde{A}_\lambda)(x)=\inf_{\lambda\in\Lambda}\tilde{A}_\lambda(x).$$

定义 1.4[6] 对于任意的 $\tilde{A},\tilde{B}\in I^X$,$\tilde{A}+\tilde{B}\in I^X$ 定义如下

$$(\tilde{A}+\tilde{B})(x)=\vee\{\tilde{A}(s)\wedge\tilde{B}(t):s+t=x\}.$$

定义 1.5[6] 对于任意的 $\tilde{A}\in I^X$,$t\in K$,$t\tilde{A}\in I^X$ 定义如下

$$(t\tilde{A})(x)=\tilde{A}(x/t),\quad t\neq 0,$$

$$(0\tilde{A})(x)=\begin{cases}\bigvee_{z\in X}\tilde{A}(z), & x=\theta,\\ 0, & x\neq\theta.\end{cases}$$

特别地,$\forall x_\lambda,y_\mu\in X$,$t\in K$,有 $x_\lambda+y_\mu=(x+y)_{\lambda\wedge\mu}$,$tx_\lambda=(tx)_\lambda$.

2 I-fuzzy 凸的相关性质

定义 2.1 设 X 是数域 K 上的线性空间,若 I-fuzzy 拓扑空间 $(X,\mathcal{T})$ 满足以下条件:

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(L_1) $\forall x_\lambda, y_\mu \in Pt(I^X), \tilde{A} \in \mathscr{T}(X)$,

$\models (\tilde{A} \in Q_{x_\lambda + y_\mu}) \to (\exists \tilde{B})(\exists \tilde{C})((\tilde{B} \in Q_{x_\lambda}) \wedge (\tilde{C} \in Q_{y_\mu}) \wedge (\tilde{B} + \tilde{C} \leqslant \tilde{A}))$;

(L_2) $\forall x_\lambda \in Pt(I^X), k_0 \in K, \tilde{A} \in \mathscr{T}(X)$,

$\models (\tilde{A} \in Q_{k_0 x_\lambda}) \to (\exists \delta)(\exists \tilde{B})((\tilde{B} \in Q_{x_\lambda}) \wedge (\forall k \in K)(|k - k_0| < \delta \to k\tilde{B} \leqslant \tilde{A}))$.

则称 $(X, \mathcal{T})$ 为 I-fuzzy 拓扑线性空间.

定义 2.2 设 X 是论域, $\forall \tilde{A} \in \mathscr{T}(\mathscr{T}(X))$, 若

$$\tilde{A} \in \vartheta :=(\forall x_\lambda)(\forall y_\mu)((x_\lambda \in \tilde{A}) \wedge (y_\mu \in \tilde{A}) \to (tx_\lambda + (1-t)y_\mu) \in \tilde{A})), \forall t \in [0,1]$$

则称一元模糊谓词 $\vartheta \in \mathscr{T}(\mathscr{T}(\mathscr{T}(X)))$ 为 I-fuzzy 凸.

定理 2.1 若 X 是 I-fuzzy 拓扑线性空间, $\tilde{A} \in \mathscr{T}(\mathscr{T}(X))$, $x_{\lambda_0} \in Pt(I^X)$, 则

$$\models \tilde{A} \in \vartheta \leftrightarrow x_{\lambda_0} + \tilde{A} \in \vartheta.$$

证明:

$$\begin{aligned}
[\tilde{A} \in \vartheta] &= \inf_{x_\lambda, y_\mu} \min(1, 1 - \min(\tilde{A}(x_\lambda), \tilde{A}(y_\mu)) + \tilde{A}(tx_\lambda + (1-t)y_\mu)) \\
&= \inf_{x_\lambda, y_\mu} \min(1, 1 - \min((\tilde{A} + x_{\lambda_0})(x_\lambda + x_{\lambda_0}), (\tilde{A} + x_{\lambda_0})(y_\mu + x_{\lambda_0})) + \\
&\quad (\tilde{A} + x_{\lambda_0})(t(x_\lambda + x_{\lambda_0}) + (1-t)(y_\mu + x_{\lambda_0}))) \\
&= [\tilde{A} + x_{\lambda_0}) \in \vartheta].
\end{aligned}$$

定理 2.2 若 X 是 I-fuzzy 拓扑线性空间, 对于任意的 $a_1, a_2 \in \mathbf{R}_+, \tilde{A} \in \mathscr{T}(\mathscr{T}(X))$, 则

$$\models \tilde{A} \in \vartheta \leftrightarrow (a_1 + a_2)\tilde{A} \equiv a_1\tilde{A} + a_2\tilde{A}.$$

证明:

$$\begin{aligned}
&[(a_1 + a_2)\tilde{A} \equiv a_1\tilde{A} + a_2\tilde{A}] \\
&= [((a_1 + a_2)\tilde{A} \leqslant a_1\tilde{A} + a_2\tilde{A}) \wedge ((a_1 + a_2)\tilde{A} \geqslant a_1\tilde{A} + a_2\tilde{A})] \\
&= \min(\inf_{z_\lambda} \min(1, 1 - (a_1 + a_2)\tilde{A}(z_\lambda) + (a_1\tilde{A} + a_2\tilde{A})(z_\lambda)), \inf_{z_\lambda} \min(1, 1 - (a_1\tilde{A} + a_2\tilde{A})(z_\lambda) + (a_1 + a_2)\tilde{A}(z_\lambda))) \\
&= \min \inf_{z_\lambda} \min(1, 1 - |(a_1 + a_2)\tilde{A}(z_\lambda) - (a_1\tilde{A} + a_2\tilde{A})(z_\lambda)|) \\
&= \inf_{z_\lambda} (1 - |(a_1 + a_2)\tilde{A}(z_\lambda) - (a_1\tilde{A} + a_2\tilde{A})(z_\lambda)|) \\
&= \inf_{z_\lambda} (1 - |\tilde{A}\left(\frac{1}{a_1 + a_2} z_\lambda\right) - \sup_{a_1 x_\mu + a_2 y_\gamma = z_\lambda} \min(a_1\tilde{A}(a_1 x_\mu), a_2\tilde{A}(a_2 y_\gamma))|) \\
&= \inf_{z_\lambda} (1 - |\tilde{A}\left(\frac{1}{a_1 + a_2} z_\lambda\right) - \sup_{a_1 x_\mu + a_2 y_\gamma = z_\lambda} \min(\tilde{A}(x_\mu), \tilde{A}(y_\gamma))|) \\
&= \inf_{z_\lambda} (1 - \sup_{a_1 x_\mu + a_2 y_\gamma = z_\lambda} |\tilde{A}\left(\frac{a_1}{a_1 + a_2} x_\mu + \frac{a_2}{a_1 + a_2} y_\gamma\right) - \min(\tilde{A}(x_\mu), \tilde{A}(y_\gamma))|) \\
&= \inf_{x_\mu, y_\gamma \in \tilde{A}} (1 - |\min(\tilde{A}(x_\mu), \tilde{A}(y_\gamma)) - \tilde{A}(tx_\mu + (1-t)y_\gamma)|) \\
&= [\tilde{A} \in \vartheta]\left(t = \frac{a_1}{a_1 + a_2}\right).
\end{aligned}$$

定理 2.3 若 X 是 I-fuzzy 拓扑线性空间, $\tilde{A}, \tilde{B}, \tilde{A}_\tau \in \mathscr{T}(\mathscr{T}(X))(\tau \in T)$, 则

(1) $\models \tilde{A} \in \vartheta \to a\tilde{A} \in \vartheta, \forall a \in \mathbf{R}_+$;

(2) $\models (\tilde{A} \in \vartheta) \wedge (\tilde{B} \in \vartheta) \to (\tilde{A} + \tilde{B}) \in \vartheta$;

(3) $\models (\forall k)((k \in \Lambda) \to \vartheta(\tilde{A}_k)) \to \vartheta(\bigcap_{k \in \Lambda} \tilde{A}_k)$.

证明:

(1)
$$\begin{aligned}
[a\tilde{A} \in \vartheta] &= \inf_{x_\lambda, y_\mu} \min(1, 1 - \min((a\tilde{A})(x_\lambda), (a\tilde{A})(y_\mu)) + (a\tilde{A})(tx_\lambda + (1-t)y_\mu)) \\
&= \inf_{x_\lambda, y_\mu} \min\left(1, 1 - \min\left(\tilde{A}\left(\frac{1}{a}x_\lambda\right), \tilde{A}\left(\frac{1}{a}y_\mu\right)\right) + \tilde{A}\left(\frac{t}{a}x_\lambda + \frac{1-t}{a}y_\mu\right)\right) \\
&\geqslant \inf_{x_\lambda, y_\mu} \min(1, 1 - \min(\tilde{A}(x_\lambda), \tilde{A}(y_\mu)) + \tilde{A}(tx_\lambda + (1-t)y_\mu)) \\
&= [\tilde{A} \in \vartheta].
\end{aligned}$$

(2)
$$\begin{aligned}
&[\tilde{A} + \tilde{B}) \in \vartheta] \\
&= \inf_{x_{\lambda_1}, y_{\lambda_2}} \min(1, 1 - \min((\tilde{A} + \tilde{B})(x_{\lambda_1}), (\tilde{A} + \tilde{B})(y_{\lambda_2})) + (\tilde{A} + \tilde{B})(tx_{\lambda_1} + (1-t)y_{\lambda_2})) \\
&= \inf_{x_{\lambda_1}, y_{\lambda_2}} \min(1, 1 - \min(\sup_{u_{\lambda_3} + v_{\lambda_4} = x_{\lambda_1}} \min(\tilde{A}(u_{\lambda_3}), \tilde{B}(v_{\lambda_4})), \sup_{z_{\lambda_5} + w_{\lambda_6} = y_{\lambda_2}} \min(\tilde{A}(z_{\lambda_5}), \tilde{B}(w_{\lambda_6})) + \\
&\quad \sup_{\substack{u_{\lambda_3} + v_{\lambda_4} = x_{\lambda_1}, \\ z_{\lambda_5} + w_{\lambda_6} = y_{\lambda_2}}} (\tilde{A} + \tilde{B})(tu_{\lambda_3} + (1-t)z_{\lambda_5} + tv_{\lambda_4} + (1-t)w_{\lambda_6})) \\
&\geqslant \min(1, 1 - \sup_{u_{\lambda_3}, v_{\lambda_4}, z_{\lambda_5}, w_{\lambda_6}} \min(\tilde{A}(u_{\lambda_3}) \wedge \tilde{A}(z_{\lambda_5}), \tilde{B}(v_{\lambda_4}) \wedge \tilde{B}(w_{\lambda_6})) + \sup_{u_{\lambda_3}, v_{\lambda_4}, z_{\lambda_5}, w_{\lambda_6}} \min(\tilde{A}(tu_{\lambda_3} + (1-t)z_{\lambda_5}), \tilde{B}(tv_{\lambda_4} + (1-t)w_{\lambda_6}))) \\
&\geqslant \min(\inf_{u_{\lambda_3}, z_{\lambda_5}} \min(1, 1 - \min(\tilde{A}(u_{\lambda_3}), \tilde{A}(z_{\lambda_5})) + \tilde{A}(tu_{\lambda_3} + (1-t)z_{\lambda_5}), \\
&\quad \inf_{v_{\lambda_4}, w_{\lambda_6}} \min(1, 1 - \min(\tilde{B}(v_{\lambda_4}), \tilde{B}(w_{\lambda_6})) + \tilde{B}(tv_{\lambda_4} + (1-t)w_{\lambda_6}))) \\
&= [\tilde{A} \in \vartheta) \wedge (\tilde{B} \in \vartheta)](t \in [0,1]).
\end{aligned}$$

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(3) $[\vartheta(\bigcap_{k\in\Lambda}\tilde{A}_k)]$

$$= \inf_{x_\lambda,y_\mu}\min(1,1-\min((\bigcap_{k\in\Lambda}\tilde{A}_k)(x_\lambda),(\bigcap_{k\in\Lambda}\tilde{A}_k)(y_\mu)) + (\bigcap_{k\in\Lambda}\tilde{A}_k)(tx_\lambda+(1-t)y_\mu))$$

$$= \inf_{x_\lambda,y_\mu}\min(1,1-\min(\inf_{k\in\Lambda}\tilde{A}_k(x_\lambda),\inf_{k\in\Lambda}\tilde{A}_k(y_\mu)) + \inf_{k\in\Lambda}\tilde{A}_k(tx_\lambda+(1-t)y_\mu))$$

$$\geqslant \inf_{x_\lambda,y_\mu}\min(1,1-\inf_{k\in\Lambda}\min(\tilde{A}_k(x_\lambda),\tilde{A}_k(y_\mu)) + \inf_{k\in\Lambda}\tilde{A}_k(tx_\lambda+(1-t)y_\mu))$$

$$\geqslant \inf_{k\in\Lambda}\inf_{x_\lambda,y_\mu}\min(1,1-\min(\tilde{A}_k(x_\lambda),\tilde{A}_k(y_\mu)) + \tilde{A}_k(tx_\lambda+(1-t)y_\mu))$$

$$= \inf_{k\in\Lambda}[\vartheta(\tilde{A}_k)] = [\forall k)(k\in\Lambda\to\vartheta(\tilde{A}_k))].$$

定理 2.4　若 X,Y 是两个 I-fuzzy 线性空间，$\tilde{A},\tilde{B}\in\mathscr{T}(X\times Y)$，且 $\tilde{C}=\{(x_{\lambda_1},y_{\lambda_2})\mid x\in X,y\in Y,(x_{\lambda_1},u_{\lambda_3})\in\tilde{A},(x_{\lambda_1},v_{\lambda_4})\in\tilde{B},u_{\lambda_3}+v_{\lambda_4}=y_{\lambda_2}\}$，则

$$\vdash(\tilde{A}\in\vartheta)\wedge(\tilde{B}\in\vartheta)\to\tilde{C}\in\vartheta.$$

证明: $[\tilde{C}\in\vartheta]$

$$= \inf_{u_{\lambda_3},v_{\lambda_4},z_{\lambda_5},w_{\lambda_6}}\min(1,1-\min(\tilde{C}(z_{\lambda_5},u_{\lambda_3}),\tilde{C}(w_{\lambda_6},v_{\lambda_4})) + \tilde{C}(t(z_{\lambda_5},u_{\lambda_3})+(1-t)(w_{\lambda_6},v_{\lambda_4}))$$

$$= \inf_{u_{\lambda_3},v_{\lambda_4},z_{\lambda_5},w_{\lambda_6}}\min(1,1-\min(\tilde{C}(z_{\lambda_5},a_{\lambda_7}+b_{\lambda_8}),\tilde{C}(w_{\lambda_6},c_{\lambda_9}+d_{\lambda_{10}})) + \tilde{C}(t(z_{\lambda_5},a_{\lambda_7}+b_{\lambda_8}) + (1-t)(w_{\lambda_6},c_{\lambda_9}+d_{\lambda_{10}})))$$

$$= \inf_{u_{\lambda_3},v_{\lambda_4},z_{\lambda_5},w_{\lambda_6}}\min(1,1-\min(\tilde{A}(z_{\lambda_5},a_{\lambda_7})\wedge\tilde{B}(z_{\lambda_5},b_{\lambda_8}),\tilde{A}(w_{\lambda_6},c_{\lambda_9})\wedge\tilde{B}(w_{\lambda_6},d_{\lambda_{10}})) + \tilde{A}(tz_{\lambda_5}+(1-t)w_{\lambda_6},ta_{\lambda_7}+(1-t)c_{\lambda_9})\wedge\tilde{B}(tz_{\lambda_5}+(1-t)w_{\lambda_6},tb_{\lambda_8}+(1-t)d_{\lambda_{10}}))$$

$$\geqslant \min(\inf_{u_{\lambda_3},v_{\lambda_4},z_{\lambda_5},w_{\lambda_6}}\min(1,1-\min(\tilde{A}(z_{\lambda_5},a_{\lambda_7}),\tilde{A}(w_{\lambda_6},c_{\lambda_9})) + \tilde{A}(tz_{\lambda_5}+(1-t)w_{\lambda_6},ta_{\lambda_7}+(1-t)c_{\lambda_9})),\ \inf_{u_{\lambda_3},v_{\lambda_4},z_{\lambda_5},w_{\lambda_6}}\min(1,1-\min(\tilde{B}(z_{\lambda_5},b_{\lambda_8}),\tilde{B}(w_{\lambda_6},d_{\lambda_{10}})) + \tilde{B}(tz_{\lambda_5}+(1-t)w_{\lambda_6},tb_{\lambda_8}+(1-t)d_{\lambda_{10}})))$$

$$= \min(\inf_{u_{\lambda_3},v_{\lambda_4},z_{\lambda_5},w_{\lambda_6}}\min(1,1-\min(\tilde{A}(z_{\lambda_5},a_{\lambda_7}),\tilde{A}(w_{\lambda_6},c_{\lambda_9})) + \tilde{A}(t(z_{\lambda_5},a_{\lambda_7})+(1-t)(w_{\lambda_6},c_{\lambda_9}))),\ \inf_{u_{\lambda_3},v_{\lambda_4},z_{\lambda_5},w_{\lambda_6}}\min(1,1-\min(\tilde{B}(z_{\lambda_5},b_{\lambda_8}),\tilde{B}(w_{\lambda_6},d_{\lambda_{10}})) + \tilde{B}(t(z_{\lambda_5},b_{\lambda_8})+(1-t)(w_{\lambda_6},d_{\lambda_{10}}))))$$

$$= [\tilde{A}\in\vartheta)\wedge(\tilde{B}\in\vartheta)].$$

定理 2.5　若 X,Y 是两个 I-fuzzy 线性空间，$f^{\rightarrow}:I^X\to I^Y$ 是线性映射，$\forall\tilde{A}\in\mathscr{T}(\mathscr{T}(X))$，$\tilde{B}\in\mathscr{T}(\mathscr{T}(Y))$，则 (1) $\vdash\vartheta(\tilde{A})\to\vartheta(f^{\rightarrow}(\tilde{A}))$；(2) $\vdash\vartheta(\tilde{B})\leftrightarrow\vartheta(f^{\leftarrow}(\tilde{B}))$.

证明: (1) $[\vartheta(f^{\rightarrow}(\tilde{A}))]$

$$= \inf_{x_{\lambda_1},y_{\lambda_2}\in Pt(I^Y)}\min(1,1-\min(f^{\rightarrow}(\tilde{A})(x_{\lambda_1}),f^{\rightarrow}(\tilde{A})(y_{\lambda_2})) + f^{\rightarrow}(\tilde{A})(tx_{\lambda_1}+(1-t)y_{\lambda_2}))$$

$$= \inf_{x_{\lambda_1},y_{\lambda_2}\in Pt(I^Y)}\min(1,1-\min(\sup_{u_{\lambda_3}\in f^{-1}(x_{\lambda_1})}\tilde{A}(u_{\lambda_3}),\sup_{v_{\lambda_4}\in f^{-1}(y_{\lambda_2})}\tilde{A}(v_{\lambda_4})) + \sup_{\substack{u_{\lambda_3}\in f^{-1}(x_{\lambda_1}),\\ v_{\lambda_4}\in f^{-1}(y_{\lambda_2})}}\tilde{A}(tu_{\lambda_3}+(1-t)v_{\lambda_4}))$$

$$\geqslant \inf_{x_{\lambda_1},y_{\lambda_2}\in Pt(I^Y)}\inf_{\substack{u_{\lambda_3}\in f^{-1}(x_{\lambda_1}),\\ v_{\lambda_4}\in f^{-1}(y_{\lambda_2})}}\min(1,1-\min(\tilde{A}(u_{\lambda_3}),\tilde{A}(v_{\lambda_4})) + \tilde{A}(tu_{\lambda_3}+(1-t)v_{\lambda_4})) = [\tilde{A}\in\vartheta].$$

(2) $[\vartheta(f^{\leftarrow}(\tilde{B}))]$

$$= \inf_{u_{\lambda_3},v_{\lambda_4}\in Pt(I^X)}\min(1,1-\min(f^{\leftarrow}(\tilde{B})(u_{\lambda_3}),f^{\leftarrow}(\tilde{B})(v_{\lambda_4})) + f^{\leftarrow}(\tilde{B})(tu_{\lambda_3}+(1-t)v_{\lambda_4}))$$

$$= \inf_{u_{\lambda_3},v_{\lambda_4}\in Pt(I^X)}\min(1,1-\min(\tilde{B}(f(u_{\lambda_3})),\tilde{B}(f(v_{\lambda_4}))) + \tilde{B}(f(tu_{\lambda_3}+(1-t)v_{\lambda_4})))$$

$$= \inf_{x_{\lambda_1},y_{\lambda_2}\in Pt(I^Y)}\min(1,1-\min(\tilde{B}(x_{\lambda_1}),\tilde{B}(y_{\lambda_2})) + \tilde{B}(tx_{\lambda_1}+(1-t)y_{\lambda_2})) = [\vartheta(\tilde{B})].$$

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Convexity in I-fuzzy Topological Linear Spaces

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Abstract: Fuzzifying topological linear spaces is generalized,the concept of I-fuzzy topological linear spaces is introduced. Furthermore,I-fuzzy convexity and related properties are also discussed.

Keywords: topological linear spaces; I-fuzzy convexity; fuzzy logic; quasi-coincident neighborhood

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