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时间尺度上一类 Leslie-Gower 捕食模型持久性及概周期解的存在性

李 玲 李建祥

(保山学院数学学院,云南 保山 678000)

摘 要

本文考虑了一类在时间尺度上具反馈项和 Holling-type Ⅱ类功能性反应的修正 Leslie-Gower 捕食模型. 通过利用 Lyapunov 函数和微分不等式,得到确保该模型持久和存在概周期解的充分条件.

关键词: 持久性,概周期解,Holling-type,修正 Leslie-Gower,时间尺度.

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1 问题背景

近年来,许多专家和学者研究了具有 Holling-type Ⅱ类功能性反应的修正 Leslie-Gower 捕食模型

$$\begin{cases} x'(t) = x(t)\left(r_1(t) - b(t)x(t) - \dfrac{a_1(t)y(t)}{x(t)+k_1}\right), \\ y'(t) = y(t)\left(r_2(t) - \dfrac{a_2(t)y(t)}{x(t)+k_2}\right), \end{cases}$$

其中,$x(t)$ 表示 t 时刻食饵总数,$y(t)$ 表示 t 时刻的捕食者总数,$r_i(t)$,$a_i(t)$,$b(t)$ $(i=1,2)$ 都是为非负连续的周期函数,$k_i(i=1,2)$ 为常数,具体的生物含义见文献[1]. 例如文献[1]利用线性周期脉冲方程中的 Floquet 理论研究了上述模型的动力学行为. 文献[2]利用重合度理论和通过建立合适的 Lyapunov 函数得到该系统正解的存在性和全局吸引性等.

1990 年,德国数学家 Stefan Hilger 在他的博士论文中建立了时标理论,即一种连续和离散计算的统一方法. 考虑到概周期现象较周期现象更为常见、自然界会受到人类的开采等因素的影响以及非自治系统更能精确地描述实际情况,本文研究时标上具反馈项和 Holling-type Ⅱ类功能性反应的修正 Leslie-Gower 捕食模型

$$\begin{cases} x^{\Delta}(t) = r_1(t) - b(t)e^{x(t)} - \dfrac{a_1(t)e^{y(t)}}{e^{x(t)}+k_1(t)} - c_1(t)u(t), \\ y^{\Delta}(t) = r_2(t) - \dfrac{a_2(t)e^{y(t)}}{e^{x(t)}+k_2(t)} - c_2(t)v(t), \\ u^{\Delta}(t) = d_1(t) - \alpha_1(t)u(t) + \beta_1(t)e^{x(t)}, \\ v^{\Delta}(t) = d_2(t) - \alpha_2(t)v(t) + \beta_2(t)e^{y(t)}, \end{cases} \tag{1.1}$$

其中,$t \in T$,T 为概周期时标,$x(t)$ 表示 t 时刻食饵总数,$y(t)$ 表示 t 时刻的捕食者总数,$u(t)$、$v(t)$ 为反馈控制项,$r_i(t)$、$a_i(t)$、$b(t)$、$c_i(t)$、$d_i(t)$、$k_i(t)$、$\alpha_i(t)$、$\beta_i(t)$ $(i=1,2)$ 为时标 T 上的非负有界概周期函数. 从生态学的角度来看,假设系统(1.1)满足初始条件:$x(0)>0$,$y(0)>0$,$u(0)>0$,$v(0)>0$.

注 1.1 令 $N(t)=e^{x(t)}$,$M(t)=e^{y(t)}$.

当 $T=R$ 时,系统(1.1)具有如下连续形式:

$$\begin{cases} N'(t) = N(t)\left(r_1(t) - b(t)N(t) - \dfrac{a_1(t)M(t)}{N(t)+k_1(t)} - c_1(t)u(t)\right), \\ M'(t) = M(t)\left(r_2(t) - \dfrac{a_2(t)M(t)}{N(t)+k_2(t)} - c_2(t)v(t)\right), \\ u'(t) = d_1(t) - \alpha_1(t)u(t) + \beta_1(t)N(t), \\ v'(t) = d_2(t) - \alpha_2(t)v(t) + \beta_2(t)M(t). \end{cases}$$

当 $T=Z$ 时,系统(1.1)具有如下离散形式:

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$$\begin{cases} N(t+1) = N(t)\,\mathrm{e}^{\left(r_1(t)-b(t)N(t)-\frac{a_1(t)M(t)}{N(t)+k_1(t)}-c_1(t)u(t)\right)}, \\ M(t+1) = M(t)\,\mathrm{e}^{\left(r_2(t)-\frac{a_2(t)M(t)}{N(t)+k_2(t)}-c_2(t)v(t)\right)}, \\ \Delta u(t) = d_1(t) - \alpha_1(t)u(t) + \beta_1(t)N(t), \\ \Delta v(t) = d_2(t) - \alpha_2(t)v(t) + \beta_2(t)M(t). \end{cases}$$

据我们所知,还没有文献讨论过系统(1.1)的持久性和存在概周期解的问题. 本文主要目的是利用 Lyapunov 函数和微分不等式,得到该系统持久和存在概周期解的充分条件.

通篇假设:

(H_1) $r_i(t)$、$a_i(t)$、$b(t)$、$c_i(t)$、$d_i(t)$、$k_i(t)$、$\alpha_i(t)$、$\beta_i(t)$ $(i=1,2)$ 为时标 T 上的非负有界概周期函数,即 $0 \leqslant r_i^m \leqslant r_i(t) \leqslant r_i^M$, $0 \leqslant a_i^m \leqslant a_i(t) \leqslant a_i^M$, $0 \leqslant b^m \leqslant b(t) \leqslant b^M$, $0 \leqslant c_i^m \leqslant c_i(t) \leqslant c_i^M$, $0 \leqslant d_i^m \leqslant d_i(t) \leqslant d_i^M$, $0 \leqslant k_i^m \leqslant k_i(t) \leqslant k_i^M$, $0 \leqslant \alpha_i^m \leqslant \alpha_i(t) \leqslant \alpha_i^M$, $0 \leqslant \beta_i^m \leqslant \beta_i(t) \leqslant \beta_i^M$, $i=1,2$ 且 $-b^M, -\alpha_i^M \in \mathbf{R}^+$.

这里,$f^m = \inf\limits_{t\in T}\{f(t)\}$, $f^M = \sup\limits_{t\in T}\{f(t)\}$.

2 预备知识及引理

定义 2.1 [5] 称时间尺度 T 是概周期的,如果 $\Pi := \{\tau \in R: t \pm \tau \in T, \forall\, t \in T\} \neq \{0\}$.

定义 2.2 [5] 令 T 为一概周期时间尺度. 称函数 $f \in C(T, \mathrm{E}^n)$ 是 T 上概周期的,如果对于任意给定的 $\varepsilon > 0$,集合 $E(\varepsilon, f) = \{\tau \in \Pi: |f(t+\tau) - f(t)| < \varepsilon, \forall\, t \in T\}$ 是相对稠密的,即对任给的 $\varepsilon > 0$,存在 $l(\varepsilon) > 0$ 使得每个长度为 $l(\varepsilon)$ 的区间内至少包含一个 $\tau(\varepsilon) \in E(\varepsilon, f)$ 使得 $|f(t+\tau) - f(t)| < \varepsilon, \forall\, t \in T$. 集合 $E(\varepsilon, f)$ 称为 $f(t)$ 的 ε - 移位数集,称为 $f(t)$ 的 ε - 移位数,$l(\varepsilon) > 0$ 称为 $E(\varepsilon, f)$ 的包含区间长.

定义 2.3 [4] 称系统(1.1)具有持久性,若存在独立于系统(1.1)的解的正常数 m_i, M_i $(i=1,2,3,4)$,使得系统(1.1)的任意解 $(x(t), y(t), u(t), v(t))^T$ 都满足

$$m_1 \leqslant \liminf_{t\to\infty} x(t) \leqslant \limsup_{t\to\infty} x(t) \leqslant M_1,$$
$$m_2 \leqslant \liminf_{t\to\infty} y(t) \leqslant \limsup_{t\to\infty} y(t) \leqslant M_2,$$
$$m_3 \leqslant \liminf_{t\to\infty} u(t) \leqslant \limsup_{t\to\infty} u(t) \leqslant M_3,$$
$$m_4 \leqslant \liminf_{t\to\infty} v(t) \leqslant \limsup_{t\to\infty} v(t) \leqslant M_4.$$

引理 2.1 [5] 设 $-a \in \mathbf{R}^+$.

(1) 若 $x^\Delta(t) \leqslant b - ax(t)$,则 $x(t) \leqslant x(t_0)\,e_{-a}(t, t_0) + \frac{b}{a}(1 - e_{-a}(t, t_0))$.

特别地,若 $a > 0$,则当 $t > t_0$ 时有 $\limsup\limits_{t\to+\infty} x(t) \leqslant \frac{b}{a}$.

(2) 若 $x^\Delta(t) \geqslant b - ax(t)$,则 $x(t) \geqslant x(t_0)\,e_{-a}(t, t_0) + \frac{b}{a}(1 - e_{-a}(t, t_0))$.

特别地,若 $a > 0$,则当 $t > t_0$ 时有 $\liminf\limits_{t\to+\infty} x(t) \geqslant \frac{b}{a}$.

引理 2.2 [5] 假设在 $T \times S \times S$ 上存在 Lyapunov 函数 $V(t, X, Y)$ 满足下面的条件:

$a(\|X - Y\|_0) \leqslant V(t, X, Y) \leqslant b(\|X - Y\|_0)$,其中 $a, b \in K$, $K = \{a \in C(\mathbf{R}^+, \mathbf{R}^+): a(0) = 0$,且 a 是增函数$\}$;

(2) $|V(t, X, Y) - V(t, X_1, Y_1)| \leqslant L(\|X - X_1\|_0 + \|Y - Y_1\|_0)$,其中 $L > 0$ 是一个常数;

(3) $D^+ V^\Delta(t, X, Y) \leqslant -cV(t, X, Y)$,其中 $-c \in \Re^+$, $c > 0$;

此外,如果系统(1.1)存在一个解 $(x(t), y(t), u(t), v(t))^T \in S$,其中 S 是一个紧集,则系统(1.1)有唯一的概周期解,且此概周期解还是一致渐近稳定的.

3 持久性

在这一节,利用不等式放缩,我们将研究系统(1.1)的持久性.

定理 3.1 假设条件 (H_1) 成立,则系统(1.1)的解 $(x(t), y(t), u(t), v(t))^T$ 满足 $\limsup\limits_{t\to\infty} x(t) \leqslant x^*$, $\limsup\limits_{t\to\infty} y(t) \leqslant y^*$, $\limsup\limits_{t\to\infty} u(t) \leqslant u^*$, $\limsup\limits_{t\to\infty} v(t) \leqslant v^*$,其中 $x^* = \frac{r_1^M}{b^m}$, $y^* = \frac{r_2^M(\mathrm{e}^{x^*} + k_2^M)}{a_2^m}$, $u^* = \frac{\beta_1^M \mathrm{e}^{x^*} + d_1^M}{\alpha_1^m}$, $v^* = \frac{\beta_2^M \mathrm{e}^{y^*} + d_2^M}{\alpha_2^m}$.

证明: 假设 $(x(t), y(t), u(t), v(t))^T$ 为系统(1.1)的解,由系统(1.1)的第一个方程得,$x^\Delta(t) = r_1(t) - b(t)\,\mathrm{e}^{x(t)} - \frac{a_1(t)\,\mathrm{e}^{y(t)}}{\mathrm{e}^{x(t)} + k_1(t)} - c_1(t)u(t) \leqslant r_1^M - b^m x(t)$. 则由引理 2.1 的结果,易知 $\limsup\limits_{t\to\infty} x(t) \leqslant \frac{r_1^M}{b^m} = x^*$.

从上式可知,对任给的 $\varepsilon > 0$,存在 $t_1 \in T$,使得 $x(t) \leqslant x^* + \varepsilon$,对于 $t > t_1$ 都成立. 因此,当 $t > t_1$ 时

$$u^\Delta(t) = d_1(t) - \alpha_1(t)u(t) + \beta_1(t)\,\mathrm{e}^{x(t)} \leqslant$$

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$$-\alpha_1^m u(t) + (d_1^M + \beta_1^M e^{(x^*+\varepsilon)}).$$

由引理 2.1 得, $\limsup_{t\to\infty} u(t) \leqslant \frac{\beta_1^M e^{x^*+\varepsilon} + d_1^M}{\alpha_1^m}$.

令上述不等式中的 $\varepsilon \to 0$, 则有 $\limsup_{t\to\infty} u(t) \leqslant \frac{\beta_1^M e^{x^*} + d_1^M}{\alpha_1^m} = u^*$.

同理可得, $\limsup_{t\to\infty} y(t) \leqslant \frac{r_2^M(e^{x^*} + k_2^M)}{a_2^m} = y^*$, $\limsup_{t\to\infty} v(t) \leqslant \frac{\beta_2^M e^{y^*} + d_2^M}{\alpha_2^m} = v^*$.

定理 3.2　假设条件 (H_1) 成立, 进一步假设 (H_2) $r_1^m - \frac{a_1^M e^{y^*}}{k_1^m} - c_1^M u^* > b^M$, $r_2^m - c_2^M v^* > \frac{a_2^M}{k_2^m}$ 成立, 则系统(1.1) 的解 $(x(t), y(t), u(t), v(t))^T$ 满足 $\liminf_{t\to\infty} x(t) \geqslant x_*$, $\liminf_{t\to\infty} y(t) \geqslant y_*$, $\liminf_{t\to\infty} u(t) \geqslant u_*$, $\liminf_{t\to\infty} v(t) \geqslant v_*$, 其中 $x_* = \ln\left(\frac{r_1^m - \frac{a_1^M e^{y^*}}{k_1^m} - c_1^m u^*}{b^M}\right)$, $y_* = \ln\left(\frac{k_2^m(r_2^m - c_2^M v^*)}{a_2^M}\right)$, $u_* = \frac{\beta_1^m e^{x_*} + d_1^m}{\alpha_1^M}$, $v_* = \frac{\beta_2^m e^{y_*} + d_2^m}{\alpha_2^M}$.

证明: 假设 $(x(t), y(t), u(t), v(t))^T$ 为系统(1.1) 的解. 对任意 $\varepsilon > 0$, 由定理 3.1 知, 存在 $t_2 \in \mathrm{T}$, 使得 $x(t) \leqslant x^* + \varepsilon$, $y(t) \leqslant y^* + \varepsilon$, $u(t) \leqslant u^* + \varepsilon$, $v(t) \leqslant v^* + \varepsilon$ 对于 $t > t_2$ 都成立.

由系统(1.1) 的第一个方程和条件 (H_1) 得,

$$x^{\Delta}(t) = r_1(t) - b(t)e^{x(t)} - \frac{a_1(t)e^{y(t)}}{e^{x(t)} + k_1(t)} - c_1(t)u(t) \geqslant r_1^m - c_1^M(u^* + \varepsilon) - \frac{a_1^M e^{y^*+\varepsilon}}{k_1^m} - b^M e^{x(t)}$$

则 $r_1^m - c_1^M(u^* + \varepsilon) - \frac{a_1^M e^{y^*+\varepsilon}}{k_1^m} - b^M e^{x(t)} \leqslant 0, t > t_2$.

否则, 存在 $t_3 > t_2$, 使得 $r_1^m - c_1^M(u^* + \varepsilon) - \frac{a_1^M e^{y^*+\varepsilon}}{k_1^m} - b^M e^{x(t_3)} > 0$ 且 $r_1^m - c_1^M(u^* + \varepsilon) - \frac{a_1^M e^{y^*+\varepsilon}}{k_1^m} - b^M e^{x(t)} \leqslant 0, t \in [t_2, t_3)$.

从而, 有 $x(t_3) < \ln\left(\frac{r_1^m - \frac{a_1^M e^{y^*+\varepsilon}}{k_1^m} - c_1^m(u^* + \varepsilon)}{b^M}\right)$

和 $x(t) \geqslant \ln\left(\frac{r_1^m - \frac{a_1^M e^{y^*+\varepsilon}}{k_1^m} - c_1^m(u^* + \varepsilon)}{b^M}\right), t \in [t_2, t_3)$. 这就意味着, $x^{\Delta}(t_3) \leqslant 0$.

另一方面, 由条件 (H_2) 知, $x^{\Delta}(t_3) \geqslant r_1^m - c_1^M(u^* + \varepsilon) - \frac{a_1^M e^{y^*+\varepsilon}}{k_1^m} - b^M e^{x(t_3)} > 0$. 产生矛盾. 因此, 当 $t > t_2$ 时, $x(t) \geqslant \ln\left(\frac{r_1^m - \frac{a_1^M e^{y^*+\varepsilon}}{k_1^m} - c_1^m(u^* + \varepsilon)}{b^M}\right)$.

由 ε 的任意性, 令 $\varepsilon \to 0$, 则有 $\liminf_{t\to\infty} x(t) \geqslant \ln\left(\frac{r_1^m - \frac{a_1^M e^{y^*}}{k_1^m} - c_1^m u^*}{b^M}\right) = x_*$.

从上式可知, 对任给的 $\varepsilon > 0$, 存在 $t_4 \in \mathrm{T}$, 使得 $x(t) \geqslant x_* + \varepsilon$, 对于 $t > t_4$ 都成立. 则当 $t > t_4$ 时, $u^{\Delta}(t) = d_1(t) - \alpha_1(t)u(t) + \beta_1(t)e^{x(t)} \geqslant (d_1^m + \beta_1^m e^{(x_*+\varepsilon)}) - \alpha_1^M u(t)$.

由引理 2.1 得, $\liminf_{t\to\infty} u(t) \geqslant \frac{\beta_1^m e^{x_*+\varepsilon} + d_1^m}{\alpha_1^M}$.

令上述不等式中的 $\varepsilon \to 0$, 则有 $\liminf_{t\to\infty} u(t) \geqslant \frac{\beta_1^m e^{x_*} + d_1^m}{\alpha_1^M} = u_*$. 同理可得, $\liminf_{t\to\infty} y(t) \geqslant \ln\left(\frac{k_2^m(r_2^m - c_2^M v^*)}{a_2^M}\right) = y_*$, $\liminf_{t\to\infty} v(t) \geqslant \frac{\beta_2^m e^{y_*} + d_2^m}{\alpha_2^M} = v_*$.

定理 3.3　假设条件 (H_1) 和 (H_2) 成立, 则系统(1.1) 是持久的.

推论 3.1　假设条件 (H_1) 和 (H_2) 成立, 则系统(1.1) 对应的连续系统和离散系统也是持久的.

4　概周期解的存在性和稳定性

假设 Ω 是系统(1.1) 的所有解 $(x(t), y(t), u(t), v(t))^T$ 组成的集合, 对于所有 $t \in T$ 满足 $x_* \leqslant x(t) \leqslant x^*$, $y_* \leqslant y(t) \leqslant y^*$, $u_* \leqslant u(t) \leqslant u^*$, $v_* \leqslant v(t) \leqslant v^*$. 易知, Ω 是系统(1.1) 的不变集.

定理 4.1　假设条件 (H_1) 和 (H_2) 成立, 则 $\Omega \neq \varnothing$.

证明: 由于 $r_i(t), a_i(t), b(t), c_i(t), d_i(t), k_i(t), \alpha_i(t), \beta_i(t)$ $(i = 1, 2)$ 为时标上的概周期函数, 则存在序列 $\{\tau_n\}$, 当 $n \to \infty$ 时, 有 $\tau_n \to \infty$, 使得 $r_i(t + \tau_n) \to r_i(t)$, $a_i(t + \tau_n) \to a_i(t)$, $b(t + \tau_n) \to b(t)$, $d_i(t + \tau_n) \to d_i(t)$, $k_i(t + \tau_n) \to k_i(t)$, $\alpha_i(t + \tau_n) \to \alpha_i(t)$, $\beta_i(t + \tau_n) \to \beta_i(t)$, $i = 1, 2$.

由定理 3.1 和定理 3.2, 知对任给的 $\varepsilon > 0$, 存

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在 $t_0 \in T$,使得当 $t > t_0$ 时,有

$x_* + \varepsilon \leqslant x(t) \leqslant x^* + \varepsilon, y_* + \varepsilon \leqslant y(t) \leqslant y^* + \varepsilon,$

$u_* + \varepsilon \leqslant u(t) \leqslant u^* + \varepsilon, v_* + \varepsilon \leqslant v(t) \leqslant v^* + \varepsilon.$

记 $x_n(t) = x(t+\tau_n)$, $y_n(t) = y(t+\tau_n)$, $u_n(t) = u(t+\tau_n)$, $v_n(t) = v(t+\tau_n)$, $t > t_0 - \tau_n$,则对任意的正整数 m,存在函数序列 $\{x_n(t): n \geqslant m\}$, $\{y_n(t): n \geqslant m\}$, $\{u_n(t): n \geqslant m\}$, $\{v_n(t): n \geqslant m\}$,使得它们在时标 T 上的任意有限区间上有收敛子列. 为方便起见,其收敛子列依然记为 $\{x_n(t): n \geqslant m\}$, $\{y_n(t): n \geqslant m\}$, $\{u_n(t): n \geqslant m\}$, $\{v_n(t): n \geqslant m\}$.

因此,有 $\bar{x}(t)$, $\bar{y}(t)$, $\bar{u}(t)$, $\bar{v}(t)$,使得当 $n \to \infty$ 时,对 $t \in T$ 有 $x_n(t) \to \bar{x}(t)$, $y_n(t) \to \bar{y}(t)$, $u_n(t) \to \bar{u}(t)$, $v_n(t) \to \bar{v}(t)$. 显然 $(\bar{x}(t), \bar{y}(t), \bar{u}(t), \bar{v}(t))^T$ 是系统(1.1)的解,并且对 $t \in T$ 有 $x_* + \varepsilon \leqslant \bar{x}(t) \leqslant x^* + \varepsilon, y_* + \varepsilon \leqslant \bar{y}(t) \leqslant y^* + \varepsilon,$

$u_* + \varepsilon \leqslant \bar{u}(t) \leqslant u^* + \varepsilon, \quad v_* + \varepsilon \leqslant \bar{v}(t) \leqslant v^* + \varepsilon.$

由于 ε 为任意小的正数,令 $\varepsilon \to 0$ 有

$$x_* \leqslant \bar{x}(t) \leqslant x^*, y_* \leqslant \bar{y}(t) \leqslant y^*,$$

$$u_* \leqslant \bar{u}(t) \leqslant u^*, v_* \leqslant \bar{v}(t) \leqslant v^*.$$

定理 4.2 假设条件 (H_1) 和 (H_2) 成立,进一步假设 (H_3) $c > 0$, $-c \in \mathbf{R}^+$,

其中 $c = \min\{P_1 - Q_1\mu, P_2 - Q_2\mu, P_3 - Q_3\mu, P_4 - Q_4\mu\}$, $\mu = \sup_{t\in T}\mu(t)$,

$$P_1 = 2b^m e^{x_*} - c_1^m - \beta_1^M e^{x^*} - \frac{a_2^m e^{y_*}}{2k_2^M} - \frac{a_1^M e^{x^*+y^*}}{e^{x_*} k_1^m} - \frac{a_1^m e^{y_*}}{e^{x^*} + k_1^M},$$

$$P_2 = \frac{2a_2{}^m e^{y_*}}{e^{x^*} + k_2^M} - \frac{a_2^m e^{y_*}}{2k_2^M} - \frac{a_1^m e^{y_*}}{e^{x^*} + k_1^M} - c_2^m - \beta_2^M e^{y^*},$$

$$P_3 = 2\alpha_1^m - c_1^m - \beta_1^M e^{x^*}, P_4 = 2\alpha_2^m - c_2^m - \beta_2^M e^{y^*},$$

$$Q_1 = \frac{a_1^{M2} e^{2y^*}}{16k_1^{m^2}} + \frac{a_2^{M2} e^{2y^*}}{16k_2^{m^2}} + \frac{a_1^m c_1^m e^{y_*}}{2k_1^M} + \frac{a_2^M c_2^M e^{x^*+y^*}}{2k_2^m e^{x_*}} - \frac{2a_1^m b^m e^{y_*+2x_*}}{(e^{x^*} + k_1^M)^2} + \frac{a_1^M b^M e^{x^*+y^*}}{k_1^m} + \frac{a_1^{m2} e^{2y_*}}{2e^{x^*} k_1^M} + \frac{a_2^{M2} e^{x^*+2y^*}}{(e^{x_*} + k_2^m)^3} + b^{M2} e^{2x^*} + b^M c_1^M e^{x^*} + \beta_1^{M2} e^{2x^*} + \alpha_1^m \beta_1^m e^{x_*},$$

$$Q_2 = \frac{a_1^{M2} e^{2y^*}}{2k_1^m e^{x_*}} + \frac{a_2^{M2} e^{2y^*}}{2k_2^m e^{x_*}} + \frac{a_1^M c_2^M e^{y^*}}{k_1^m} + \frac{a_2^M c_2^M e^{y^*}}{k_2^m} + \frac{a_1^M b^M e^{x^*+y^*}}{k_1^m} + \frac{a_1^{m2} e^{2y_*}}{2k_1^M e^{x_*}} + \frac{a_2^{M2} e^{x^*+2y^*}}{(e^{x_*} + k_2^m)^3} + \beta_2^{M2} e^{2y^*} + \alpha_2^m \beta_2^m e^{y_*},$$

$$Q_3 = c_1^{M2} + \alpha_1^{M2} + \alpha_1^m \beta_1^m e^{x_*} + b^M c_1^M e^{x^*} + \frac{a_1^M c_1^M e^{y^*}}{k_1^m} + \frac{a_1^m c_1^m e^{y_*}}{2k_1^M},$$

$$Q_4 = c_2^{M2} + \alpha_2^{M2} + \alpha_2^m \beta_2^m e^{y_*} + \frac{a_2^M c_2^M e^{y^*}}{k_2^m} + \frac{a_2^M c_2^M e^{x^*+y^*}}{2k_2^m e^{x_*}}.$$

则系统(1.1)存在唯一的概周期解 $(x(t), y(t), u(t), v(t))^T \in \Omega$,且此概周期解是一致渐近稳定的.

证明: 由定理 4.1 知,系统(1.1)有一有界解 $(x(t), y(t), u(t), v(t))^T$ 满足

$$0 < x_* \leqslant x(t) \leqslant x^*, 0 < y_* \leqslant y(t) \leqslant y^*,$$

$$0 < u_* \leqslant u(t) \leqslant u^*, 0 < v_* \leqslant v(t) \leqslant v^*,$$

$$\forall t \in T.$$

则 $|x(t)| \leqslant A_1$, $|y(t)| \leqslant A_2$, $|u(t)| \leqslant A_3$, $|v(t)| \leqslant A_4$,其中 $A_1 = \max\{x^*, x_*\}$,

$$A_2 = \max\{y^*, y_*\}, A_3 = \max\{u^*, u_*\},$$

$$A_4 = \max\{v^*, v_*\}.$$

对任意的 $(x(t), y(t), u(t), v(t))^T \in \mathbf{R}^4$,定义范数

$$\|(x(t), y(t), u(t), v(t))^T\| = \sup_{t\in T}|x(t)| + \sup_{t\in T}|y(t)| + \sup_{t\in T}|u(t)| + \sup_{t\in T}|v(t)|.$$

假设 $X = (x(t), y(t), u(t), v(t))^T$ 和 $Y = (p(t), q(t), w(t), z(t))^T$ 是系统(1.1)的两个解,则有 $\|X\| \leqslant C$, $\|Y\| \leqslant C$,其中 $C = \sum_{i=1}^{4} A_i$.

考虑系统(1.1)的乘积系统

$$\begin{cases} x^{\Delta}(t) = r_1(t) - b(t)e^{x(t)} - \dfrac{a_1(t)e^{y(t)}}{e^{x(t)} + k_1(t)} - c_1(t)u(t), \\ y^{\Delta}(t) = r_2(t) - \dfrac{a_2(t)e^{y(t)}}{e^{x(t)} + k_2(t)} - c_2(t)v(t), \\ u^{\Delta}(t) = d_1(t) - \alpha_1(t)u(t) + \beta_1(t)e^{x(t)}, \\ v^{\Delta}(t) = d_2(t) - \alpha_2(t)v(t) + \beta_2(t)e^{y(t)}, \\ p^{\Delta}(t) = r_1(t) - b(t)e^{p(t)} - \dfrac{a_1(t)e^{q(t)}}{e^{p(t)} + k_1(t)} - c_1(t)w(t), \\ q^{\Delta}(t) = r_2(t) - \dfrac{a_2(t)e^{q(t)}}{e^{p(t)} + k_2(t)} - c_2(t)z(t), \\ w^{\Delta}(t) = d_1(t) - \alpha_1(t)w(t) + \beta_1(t)e^{p(t)}, \\ z^{\Delta}(t) = d_2(t) - \alpha_2(t)z(t) + \beta_2(t)e^{q(t)}. \end{cases} \tag{4.1}$$

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在 $T\times\Omega\times\Omega$ 上定义 Lyapunov 函数

$$V(t,X,Y)=(x(t)-p(t))^2+(y(t)-q(t))^2+(u(t)-w(t))^2+(v(t)-z(t))^2.$$

易知范数

$$\|X-Y\|_*=\sqrt{\sup_{t\in T}(x(t)-p(t))^2+\sup_{t\in T}(y(t)-q(t))^2+\sup_{t\in T}(u(t)-w(t))^2+\sup_{t\in T}(v(t)-z(t))^2}$$

等价，即存在常数 $C_1>0,C_2>0$，使得 $C_1\|X-Y\|\leqslant\|X-Y\|_*\leqslant C_2\|X-Y\|$. 因此，$(C_1\|X-Y\|)^2\leqslant V(t,X,Y)\leqslant(C_2\|X-Y\|)^2$.

令 $a(t)=C_1^2t^2,b(t)=C_2^2t^2$，则满足引理 2.2 的条件(1).

此外，对 $X_1=(x_1(t),y_1(t),u_1(t),v_1(t))^T,Y_1=(p_1(t),q_1(t),w_1(t),z_1(t))^T$ 有

$$\begin{aligned}|V(t,X,Y)-V(t,X_1,Y_1)|=&|(x(t)-p(t))^2+(y(t)-q(t))^2+(u(t)-w(t))^2+(v(t)-z(t))^2-(x_1(t)-p_1(t))^2-(y_1(t)-q_1(t))^2-(u_1(t)-w_1(t))^2-(v_1(t)-z_1(t))^2|\leqslant\\&|(x(t)-p(t))+(x_1(t)-p_1(t))|\cdot|(x(t)-p(t))-(x_1(t)-p_1(t))|+|(y(t)-q(t))+(y_1(t)-q_1(t))|\cdot|(y(t)-q(t))-(y_1(t)-q_1(t))|+\\&|(u(t)-w(t))+(u_1(t)-w_1(t))|\cdot|(u(t)-w(t))-(u_1(t)-w_1(t))|+|(v(t)-z(t))+(v_1(t)-z_1(t))|\cdot|(v(t)-z(t))-(v_1(t)-z_1(t))|\leqslant\\&L\{(|x(t)-x_1(t)|+|y(t)-y_1(t)|+|u(t)-u_1(t)|+|v(t)-v_1(t)|)+(|p(t)-p_1(t)|+|q(t)-q_1(t)|+|w(t)-w_1(t)|+|z(t)-z_1(t)|)\}=\\&L\{\|X-X_1\|+\|Y-Y_1\|\},\end{aligned}$$

其中 $L=4\max\{A_1,A_2,A_3,A_4\}$.

因此，满足引理 2.2 的条件(2).

沿系统(1.1)的解计算 $V(t,X,Y)$ 的右上导数得，

$$\begin{aligned}D^+V^\Delta(t,X,Y)=&[2(x(t)-p(t))+\mu(t)(x(t)-p(t))^\Delta](x(t)-p(t))^\Delta+[2(y(t)-q(t))+\mu(t)(y(t)-q(t))^\Delta]\cdot(y(t)-q(t))^\Delta+[2(u(t)-w(t))+\mu(t)(u(t)-w(t))^\Delta](u(t)-w(t))^\Delta+\\&[2(v(t)-z(t))+\mu(t)(v(t)-z(t))^\Delta]\cdot(v(t)-z(t))^\Delta.\end{aligned}$$

为方便起见，记 $\bar{x}(t)=x(t)-p(t),\bar{y}(t)=y(t)-q(t),\bar{u}(t)=u(t)-w(t),\bar{v}(t)=v(t)-z(t)$，则 $V(t,X,Y)=\bar{x}^2(t)+\bar{y}^2(t)+\bar{u}^2(t)+\bar{v}^2(t)$，

$$\begin{aligned}D^+V^\Delta(t,X,Y)=&(2\bar{x}(t)+\mu(t)\bar{x}^\Delta(t))\bar{x}^\Delta(t)+(2\bar{y}(t)+\mu(t)\bar{y}^\Delta(t))\bar{y}^\Delta(t)+(2\bar{u}(t)+\mu(t)\bar{u}^\Delta(t))\bar{u}^\Delta(t)+(2\bar{v}(t)+\mu(t)\bar{v}^\Delta(t))\bar{v}^\Delta(t)=\\&V_1+V_2+V_3+V_4.\end{aligned}$$

利用微分中值定理，有 $e^{x(t)}-e^{p(t)}=e^{\xi(t)}(x(t)-p(t))$，$e^{y(t)}-e^{q(t)}=e^{\eta(t)}(y(t)-q(t))$，其中 $\xi(t)$ 介于 $x(t)$ 与 $p(t)$ 之间，$\eta(t)$ 介于 $y(t)$ 与 $q(t)$ 之间.

则(4.1)可改写为

$$\begin{cases}\bar{x}^\Delta(t)=\left(\dfrac{a_1(t)e^{q(t)+\xi(t)}}{(e^{x(t)}+k_1(t))(e^{p(t)}+k_1(t))}-b(t)e^{\xi(t)}\right)\bar{x}(t)-\dfrac{a_1(t)e^{\eta(t)}}{e^{x(t)}+k_1(t)}\bar{y}(t)-c_1(t)\bar{u}(t),\\ \bar{y}^\Delta(t)=\dfrac{a_2(t)e^{q(t)+\xi(t)}}{(e^{x(t)}+k_2(t))(e^{p(t)}+k_2(t))}\bar{x}(t)-\dfrac{a_2(t)e^{\eta(t)}}{e^{x(t)}+k_2(t)}\bar{y}(t)-c_2(t)\bar{v}(t),\\ \bar{u}^\Delta(t)=-\alpha_1(t)\bar{u}(t)+\beta_1(t)e^{\xi(t)}\bar{x}(t),\\ \bar{v}^\Delta(t)=-\alpha_2(t)\bar{v}(t)+\beta_2(t)e^{\eta(t)}\bar{y}(t).\end{cases}$$

则

$$\begin{aligned}V_1=&(2\bar{x}(t)+\mu(t)\bar{x}^\Delta(t))\bar{x}^\Delta(t)\\=&\left[\left(\frac{a_1(t)e^{q(t)+\xi(t)}}{(e^{x(t)}+k_1(t))(e^{p(t)}+k_1(t))}-b(t)e^{\xi(t)}\right)^2\mu(t)+\frac{2a_1(t)e^{q(t)+\xi(t)}}{(e^{x(t)}+k_1(t))(e^{p(t)}+k_1(t))}-2b(t)e^{\xi(t)}\right]\bar{x}^2(t)+\\&\frac{a_1^2(t)e^{2\eta(t)}\mu(t)}{(e^{x(t)}+k_1(t))^2}\bar{y}^2(t)+c_1^2(t)\mu(t)\bar{u}^2(t)+\frac{2a_1(t)c_1(t)e^{\eta(t)}\mu(t)}{e^{x(t)}+k_1(t)}\bar{y}(t)\bar{u}(t)+\end{aligned}$$

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$$\left[\left(2b(t)c_1(t)\mathrm{e}^{\xi(t)}-\frac{2a_1(t)c_1(t)\mathrm{e}^{q(t)+\xi(t)}}{(\mathrm{e}^{x(t)}+k_1(t))(\mathrm{e}^{p(t)}+k_1(t))}\right)\mu(t)-2c_1(t)\right]\bar{x}(t)\bar{u}(t)+$$

$$\left[\left(\frac{2a_1(t)b(t)\mathrm{e}^{\xi(t)+\eta(t)}}{\mathrm{e}^{x(t)}+k_1(t)}-\frac{2a_1^2(t)\mathrm{e}^{q(t)+\xi(t)+\eta(t)}}{(\mathrm{e}^{x(t)}+k_1(t))^2(\mathrm{e}^{p(t)}+k_1(t))}\right)\mu(t)-\frac{2a_1(t)\mathrm{e}^{\eta(t)}}{\mathrm{e}^{x(t)}+k_1(t)}\right]\bar{x}(t)\bar{y}(t)$$

$$\leqslant\left[\left(\frac{a_1^{M^2}\mathrm{e}^{2y^*}}{16k_1^{m^2}}+b^{M^2}\mathrm{e}^{x^{*2}}-\frac{2a_1^mb^m\mathrm{e}^{y_*+2x_*}}{(\mathrm{e}^{x^*}+k_1^M)^2}\right)\mu(t)+\frac{a_1^M\mathrm{e}^{x^*+y^*}}{k_1^m\mathrm{e}^{x_*}}-2b^m\mathrm{e}^{x_*}\right]\bar{x}^2(t)+$$

$$\frac{a_1^{M^2}\mathrm{e}^{2y^*}\mu(t)}{2k_1^m\mathrm{e}^{x_*}}\bar{y}^2(t)+c_1^{M^2}\mu(t)\bar{u}^2(t)+\frac{a_1^Mc_1^M\mathrm{e}^{y^*}\mu(t)}{k_1^m}(\bar{y}^2(t)+\bar{u}^2(t))+$$

$$\left|\left(b^Mc_1^M\mathrm{e}^{x^*}-\frac{a_1^mc_1^m\mathrm{e}^{y_*+x_*}}{(\mathrm{e}^{x^*}+k_1^M)^2}\right)\mu(t)-c_1^m\right|(\bar{x}^2(t)+\bar{u}^2(t))+$$

$$\left|\left(\frac{a_1^Mb^M\mathrm{e}^{x^*+y^*}}{k_1^m}-\frac{a_1^{m^2}\mathrm{e}^{2y_*+x_*}}{(\mathrm{e}^{x^*}+k_1^M)^3}\right)\mu(t)-\frac{a_1^m\mathrm{e}^{y_*}}{\mathrm{e}^{x^*}+k_1^M}\right|(\bar{x}^2(t)+\bar{y}^2(t)).$$

$$V_2=(2\bar{y}(t)+\mu(t)\bar{y}^{\Delta}(t))\bar{y}^{\Delta}(t)$$

$$=\frac{a_2^2(t)\mathrm{e}^{2q(t)+2\xi(t)}\mu(t)}{(\mathrm{e}^{x(t)}+k_2(t))^2(\mathrm{e}^{p(t)}+k_2(t))^2}\bar{x}^2(t)+c_2^2(t)\mu(t)\bar{v}^2(t)+$$

$$\left(\frac{a_2^2(t)\mathrm{e}^{2\eta(t)}\mu(t)}{(\mathrm{e}^{x(t)}+k_2(t))^2}-\frac{2a_2(t)\mathrm{e}^{\eta(t)}}{\mathrm{e}^{x(t)}+k_2(t)}\right)\bar{y}^2(t)-\frac{2a_2(t)c_2(t)\mathrm{e}^{q(t)+\xi(t)}\mu(t)}{(\mathrm{e}^{x(t)}+k_2(t))(\mathrm{e}^{p(t)}+k_2(t))}\bar{x}(t)\bar{v}(t)+$$

$$\left(\frac{2a_2(t)\mathrm{e}^{q(t)+\xi(t)}}{(\mathrm{e}^{x(t)}+k_2(t))(\mathrm{e}^{p(t)}+k_2(t))}-\frac{2a_2^2(t)\mathrm{e}^{q(t)+\xi(t)+\eta(t)}\mu(t)}{(\mathrm{e}^{x(t)}+k_2(t))^2(\mathrm{e}^{p(t)}+k_2(t))}\right)\bar{x}(t)\bar{y}(t)+$$

$$\left(\frac{2a_2(t)c_2(t)\mathrm{e}^{\eta(t)}\mu(t)}{\mathrm{e}^{x(t)}+k_2(t)}-2c_2(t)\right)\bar{y}(t)\bar{v}(t)\leqslant$$

$$\frac{a_2^{M^2}\mathrm{e}^{2y^*}\mu(t)}{16k_2^{m^2}}\bar{x}^2(t)+\left(\frac{a_2^{M^2}\mathrm{e}^{2y^*}\mu(t)}{2k_2^m\mathrm{e}^{x_*}}-\frac{2a_2^m\mathrm{e}^{y_*}}{\mathrm{e}^{x^*}+k_2^M}\right)\bar{y}^2(t)+c_2^{M^2}\mu(t)\bar{v}^2(t)+$$

$$\frac{a_2^Mc_2^M\mathrm{e}^{x^*+y^*}\mu(t)}{2k_2^m\mathrm{e}^{x_*}}(\bar{x}^2(t)+\bar{v}^2(t))+\left|\frac{a_2^{M^2}\mathrm{e}^{2y^*}\mu(t)}{2k_2^m\mathrm{e}^{x_*}}-\frac{a_2^m\mathrm{e}^{x_*+y_*}}{2k_2^M\mathrm{e}^{x^*}}\right|(\bar{x}^2(t)+\bar{y}^2(t))+$$

$$\left|\frac{a_2^Mc_2^M\mathrm{e}^{y^*}\mu(t)}{k_2^m}-c_2^m\right|(\bar{y}^2(t)+\bar{v}^2(t)).$$

$$V_3=(2\bar{u}(t)+\mu(t)\bar{u}^{\Delta}(t))\bar{u}^{\Delta}(t)$$

$$=\beta_1^2(t)\mathrm{e}^{2\xi(t)}\mu(t)\bar{x}^2(t)+(\alpha_1^2(t)\mu(t)-2\alpha_1(t))\bar{u}^2(t)+(2\beta_1(t)\mathrm{e}^{\xi(t)}-2\alpha_1(t)\beta_1(t)\mathrm{e}^{\xi(t)}\mu(t))\bar{x}(t)\bar{u}(t)\leqslant$$

$$\beta_1^{M^2}\mathrm{e}^{2x^*}\mu(t)\bar{x}^2(t)+(\alpha_1^{M^2}\mu(t)-2\alpha_1^m)\bar{u}^2(t)+|\beta_1^M\mathrm{e}^{x^*}-\alpha_1^m\beta_1^m\mathrm{e}^{x_*}\mu(t)|(\bar{x}^2(t)+\bar{u}^2(t)).$$

$$V_4=(2\bar{v}(t)+\mu(t)\bar{v}^{\Delta}(t))\bar{v}^{\Delta}(t)$$

$$=\beta_2^2(t)\mathrm{e}^{2\eta(t)}\mu(t)\bar{y}^2(t)+(\alpha_2^2(t)\mu(t)-2\alpha_2(t))\bar{v}^2(t)+$$

$$(2\beta_2(t)\mathrm{e}^{\eta(t)}-2\alpha_2(t)\beta_2(t)\mathrm{e}^{\eta(t)}\mu(t))\bar{y}(t)\bar{v}(t)\leqslant$$

$$\beta_2^{M^2}\mathrm{e}^{2y^*}\mu(t)\bar{y}^2(t)+(\alpha_2^{M^2}\mu(t)-2\alpha_2^m)\bar{v}^2(t)+|\beta_2^M\mathrm{e}^{y^*}-\alpha_2^m\beta_2^m\mathrm{e}^{y_*}\mu(t)|(\bar{y}^2(t)+\bar{v}^2(t))$$

则有

$$D^+V^{\Delta}(t,X,Y)=V_1+V_2+V_3+V_4$$

$$\leqslant\left[\frac{a_1^{M^2}\mathrm{e}^{2y^*}}{16k_1^{m^2}}+\frac{a_2^{M^2}\mathrm{e}^{2y^*}}{16k_2^{m^2}}+\frac{a_1^mc_1^m\mathrm{e}^{y_*}}{2k_1^M}+\frac{a_2^Mc_2^M\mathrm{e}^{x^*+y^*}}{2k_2^m\mathrm{e}^{x_*}}-\frac{2a_1^mb^m\mathrm{e}^{y_*+2x_*}}{(\mathrm{e}^{x^*}+k_1^M)^2}+\frac{a_1^Mb^M\mathrm{e}^{x^*+y^*}}{k_1^m}+\frac{a_1^{m^2}\mathrm{e}^{2y_*}}{2\mathrm{e}^{x^*}k_1^M}+\right.$$

$$\left.\frac{a_2^{M^2}\mathrm{e}^{x^*+2y^*}}{(\mathrm{e}^{x_*}+k_2^m)^3}+b^{M^2}\mathrm{e}^{2x^*}+b^Mc_1^M\mathrm{e}^{x^*}+\beta_1^{M^2}\mathrm{e}^{2x^*}+\alpha_1^m\beta_1^m\mathrm{e}^{x_*}\right)\mu(t)+\frac{a_1^m\mathrm{e}^{y_*}}{\mathrm{e}^{x^*}+k_1^M}+\frac{a_1^M\mathrm{e}^{x^*+y^*}}{\mathrm{e}^{x_*}k_1^m}+\frac{a_2^m\mathrm{e}^{y_*}}{2k_2^M}+$$

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$$c_1^m + \beta_1^M e^{x^*} - 2b^m e^{x_*}\Big]\bar{x}^2(t) + \Big[\Big(\frac{a_1^{M^2} e^{2y^*}}{2k_1^m e^{x_*}} + \frac{a_2^{M^2} e^{2y^*}}{2k_2^m e^{x_*}} + \frac{a_1^M c_2^M e^{y^*}}{k_1^m} + \frac{a_2^M c_2^M e^{y^*}}{k_2^m} + \frac{a_1^M b^M e^{x^*+y^*}}{k_1^m} + \frac{a_1^{m^2} e^{2y_*}}{2k_1^M e^{x_*}} +$$

$$\frac{a_2^{M^2} e^{x^*+2y^*}}{(e^{x_*} + k_2^m)^3} + \beta_2^{M^2} e^{2y^*} + \alpha_2^m \beta_2^m e^{y_*}\Big)\mu(t) + \frac{a_1^m e^{y_*}}{e^{x^*} + k_1^M} + \frac{a_2^m e^{y_*}}{2k_2^M} - \frac{2a_2{}^m e^{y_*}}{e^{x^*} + k_2^M} + c_2^m + \beta_2^M e^{y^*}\Big]\bar{y}^2(t) +$$

$$\Big[\Big(c_1^{M^2} + \alpha_1^{M^2} + \alpha_1^m \beta_1^m e^{x_*} + b^M c_1^M e^{x^*} + \frac{a_1^M c_1^M e^{y^*}}{k_1^m} + \frac{a_1^m c_1^m e^{y_*}}{2k_1^M}\Big)\mu(t) + c_1^m + \beta_1^M e^{x^*} - 2\alpha_1^m\Big]\bar{u}^2(t) +$$

$$\Big[\Big(c_2^{M^2} + \alpha_2^{M^2} + \alpha_2^m \beta_2^m e^{y_*} + \frac{a_2^M c_2^M e^{y^*}}{k_2^m} + \frac{a_2^M c_2^M e^{x^*+y^*}}{2k_2^m e^{x_*}}\Big)\mu(t) + c_2^m + \beta_2^M e^{y^*} - 2\alpha_2^m\Big]\bar{v}^2(t)$$

$$\leqslant -[(P_1 - Q_1\mu)\bar{x}^2(t) + (P_2 - Q_2\mu)\bar{y}^2(t) + (P_3 - Q_3\mu)\bar{u}^2(t) + (P_4 - Q_4\mu)\bar{v}^2(t)]$$

$$\leqslant -c(\bar{x}^2(t) + \bar{y}^2(t) + \bar{u}^2(t) + \bar{v}^2(t)) = -cV(t, X, Y).$$

因此,满足引理 2.2 **的条件**(3). **故系统**(1.1) **有唯一的一致渐近稳定的正的概周期解.**

推论 4.1　假设条件(H_1)、(H_2) **和**(H_3) **成立,则系统**(1.1) **对应的连续系统和离散系统有唯一的正的概周期解,且这个概周期解是一致渐近稳定的.**

5　例　子

例 5.1　考虑如下系统

$$\begin{cases} x^{\Delta}(t) = r_1(t) - b(t)e^{x(t)} - \dfrac{a_1(t)e^{y(t)}}{e^{x(t)} + 5} - \\ \qquad 0.001u(t), \\ y^{\Delta}(t) = r_2(t) - \dfrac{a_2(t)e^{y(t)}}{e^{x(t)} + 4.9} - 0.001v(t), \\ u^{\Delta}(t) = -\alpha_1(t)u(t) + 0.001e^{x(t)}, \\ v^{\Delta}(t) = -\alpha_2(t)v(t) + 0.001e^{y(t)}, \end{cases}$$

其中 $r_1(t) = 1.1399 + 0.0001\sin t$, $r_1(t) = 0.4995 + 0.0005\cos(\sqrt{2}t)$,

$a_1(t) = 0.04 + 0.01\sin(\sqrt{2}t)$, $a_2(t) = 2.31 + 0.01\cos(t)$,

$\alpha_1(t) = \alpha_2(t) = 0.55 + 0.05\cos(2t)$, **令** $T = \bigcup_{k=0}^{\infty}[1.01k, 1.01k + 1]$, **通过计算可知该系统满足定理** 3.3 **和定理** 4.1 **的条件,则该系统是持久的,并且存在唯一的正的一致渐近稳定的概周期解.**

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Persistence and Almost Periodic Solutions of the Predator-Prey Model with Modified Leslie-Gower on Time Scales

Li Ling Li Jianxiang

(Mathematics College, Baoshan Univercity, Baoshan Yunnan 678000)

Abstract

In this paper, a predator-prey model with modified Leslie-Gower Holling-type Ⅱ schemes and feedback control are considered on time scales. By utilizing the Lyapunov function and the differential inequalities, some sufficient conditions are obtained for the persistence and the existence of almost periodic solutions of the system.

Key words: persistence, almost periodic solutions, Holling-type, modified Leslie-Gower, time scales.

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