# 玻色子体系的Bogoliubov变换*

白伊秀**, 胡 洁

(首都师范大学物理系, 北京 100048)

**摘要**:对于满足反对易关系的费米子体系,Bogoliubov变换是幺正变换,可以直接对体系哈密顿量进行矩阵对角化,而玻色子体系满足对易关系,不能直接进行矩阵对角化.本文以玻色子体系为研究对象,通过对哈密顿量进行改造,实现对角化,使其在自身表象下表示,并且此方法可以推广至高维自旋系统.

**关键词**:冷原子;玻色子;费米子;Bogoliubov变换;幺正变换

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## 0 引 言

在量子力学中[1],算符在不同表象中有不同的表现形式.当用矩阵表示算符时,如果取本征矢为基矢时,此算符矩阵为对角矩阵,当算符处于自身表象时,很多问题处理起来比较方便,如分析准粒子能谱、计算单分量玻色超流体系中的衰变率等[2-3].因此,需要将研究体系中的算符对角化到本征态下,对角化算符的过程称为幺正变换.在冷原子体系中,当研究低能物理中的玻色-爱因斯坦凝聚[4-6]、超流[4]和巴丁-库柏-徐瑞弗(Bardeen-Cooper-Schrieffer,BCS)超导态[7-9]时,需要做Bogoliubov变换[10-12],使其从粒子数表象变成准粒子表象,而Bogoliubov变换的本质就是对角化平均场近似下的体系哈密顿量.进行Bogoliubov变换时需要考虑体系中的粒子是玻色子还是费米子,如果体系中的粒子为费米子,则哈密顿量的对角化按正常的幺正变换进行操作,但是若为玻色子,则不能做哈密顿量的幺正变换,即量子力学中的幺正变换只适用于费米子体系,而不适用于玻色子体系,其根本原因在于幺正变换是一个三角变换($\sin^2\theta+\cos^2\theta=1$),满足反对易关系,而玻色子对角化是双曲变换($\cosh^2\theta-\sinh^2\theta=1$),需满足对易关系,因此,不能直接用幺正变换,通常可以选择代数方法,在满足玻色子对易关系的条件下对角化哈密顿量,但是由于计算的复杂性,代数方法不利于推广到高维哈密顿量体系,因此,本文研究如何在玻色子体系中通过矩阵变换实现Bogoliubov变换.

## 1 费米子体系Bogoliubov变换

在研究玻色子体系之前,首先考虑一个自旋1/2、相互作用为接触势能的费米子体系,其二次量子化之后的哈密顿量为[13-15]

$$\hat{H}^{\mathrm{F}}=\sum_{k,\sigma}\left(\varepsilon_k^{\mathrm{F}}-\mu_{\mathrm{F}}\right)\hat{c}_{k,\sigma}^{\dagger}\hat{c}_{k,\sigma}-g_{\mathrm{F}}\sum_{k,k',q}\hat{c}_{k+\frac{q}{2},\uparrow}^{\dagger}\hat{c}_{-k+\frac{q}{2},\downarrow}^{\dagger}\hat{c}_{-k'+\frac{q}{2},\downarrow}\hat{c}_{k'+\frac{q}{2},\uparrow},\tag{1}$$

式中$\varepsilon_k^{\mathrm{F}}=\hbar^2|\boldsymbol{k}|^2/(2m_{\mathrm{F}})$是费米子动能,$\mu_{\mathrm{F}}$为化学势,$\hat{c}_k^{\dagger}(\hat{c}_k)$为费米子产生(湮灭)算符,$g_{\mathrm{F}}$是费米子之间的相互作用强度.为了得到库柏对超导基态,需要对费米子做平均场理论[7-8],并代入到BCS的相互作用项$-g_{\mathrm{F}}\sum_{k,k',q}\hat{c}_{k+\frac{q}{2},\uparrow}^{\dagger}\hat{c}_{-k+\frac{q}{2},\downarrow}^{\dagger}\hat{c}_{-k'+\frac{q}{2},\downarrow}\hat{c}_{k'+\frac{q}{2},\uparrow}$中,主要关注$\boldsymbol{q}=0$的低能散射过程,令$\Delta=g_{\mathrm{F}}\sum_k\hat{c}_{-k,\downarrow}\hat{c}_{k,\uparrow}$,$\Delta$相当于"对算子"的外势场,因此,称为对势,则BCS平均场哈密顿量为

$$\hat{H}_{\mathrm{MF}}^{\mathrm{F}}=\sum_{k,\sigma}\left(\varepsilon_k^{\mathrm{F}}-\mu_{\mathrm{F}}\right)\hat{c}_{k,\sigma}^{\dagger}\hat{c}_{k,\sigma}-\sum_k\Delta\hat{c}_{k,\uparrow}^{\dagger}\hat{c}_{-k,\downarrow}^{\dagger}-\sum_k\Delta^*\hat{c}_{-k,\downarrow}\hat{c}_{k,\uparrow}+\frac{|\Delta|^2}{g_{\mathrm{F}}}.\tag{2}$$

在Nambu表象中用矩阵表示此哈密顿量为

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** 通信作者:840196742@qq.com

$$\hat{H}_{\mathrm{MF}}^{\mathrm{F}}=\sum_k \hat{\psi}_k^{\dagger}\hat{H}_k^{\mathrm{F}}\hat{\psi}_k+\text{const.}, \tag{3}$$

式中

$$\hat{\psi}_k=\left(\hat{c}_{k,\uparrow},\hat{c}_{-k,\downarrow}^{\dagger}\right)^{\mathrm{T}}, \tag{4}$$

为Nambu表象下的费米子场算符,$\hat{H}_k^{\mathrm{F}}$的矩阵形式为

$$\hat{H}_k^{\mathrm{F}}=\begin{pmatrix}\varepsilon_k^{\mathrm{F}}-\mu_{\mathrm{F}} & \Delta\\ \Delta^{*} & -\left(\varepsilon_k^{\mathrm{F}}-\mu_{\mathrm{F}}\right)\end{pmatrix}. \tag{5}$$

利用Bogoliubov变换将上式哈密顿量转至准粒子的本征表象,假设超导态的准粒子湮灭算符为$\hat{\beta}_k$,幺正变换的变换矩阵为$\hat{R}$.则超导准粒子算符$\hat{\beta}_k$与费米子算符$\hat{c}_k$之间的变换关系为

$$\begin{pmatrix}\hat{\beta}_k\\ \hat{\beta}_{-k}^{\dagger}\end{pmatrix}=\hat{R}^{\dagger}\begin{pmatrix}\hat{c}_k\\ \hat{c}_{-k}^{\dagger}\end{pmatrix}, \tag{6}$$

且准粒子表象下的矩阵哈密顿量$\hat{H}_{k,\mathrm{Bo}}^{\mathrm{F}}$与粒子数表象下的哈密顿量关系为

$$\hat{H}_{k,\mathrm{Bo}}^{\mathrm{F}}=\hat{R}^{\dagger}\hat{H}_k^{\mathrm{F}}\hat{R}. \tag{7}$$

通过解$\hat{H}_k^{\mathrm{F}}$的久期方程得到本征值$\left(\chi_{1,k},\chi_{2,k}\right)$和对应的本征态$\left(\zeta_{k,1},\zeta_{k,2}\right)$,并且用本征态构造Bogoliubov变换矩阵$\hat{R}=\left(\zeta_{k,1},\zeta_{k,2}\right)$,使得哈密顿量$\hat{H}_{\mathrm{MF}}^{\mathrm{F}}$完成对角化.

对于幺正变换而言,本征态的归一化条件为$\sum_j\left|\left(\zeta_{k,i}\right)_j\right|^2=1$,$i$表示第$i$个本征态,$j$为第$i$个本征态的第$j$个元素,根据式(5)$\hat{H}_k^{\mathrm{F}}$的久期方程,得本征值及本征态分别为

$$\chi_{1,k}=-\chi_{2,k}=\sqrt{\left(\varepsilon_k^{\mathrm{F}}-\mu_{\mathrm{F}}\right)^2+|\Delta|^2}, \tag{8}$$

$$\zeta_{k,1}=\begin{pmatrix}\sqrt{\frac{1}{2}\left(1+\frac{\varepsilon_k^{\mathrm{F}}-\mu_{\mathrm{F}}}{E_k}\right)}\\ -\sqrt{\frac{1}{2}\left(1-\frac{\varepsilon_k^{\mathrm{F}}-\mu_{\mathrm{F}}}{E_k}\right)}\end{pmatrix},$$

$$\zeta_{k,2}=\begin{pmatrix}\sqrt{\frac{1}{2}\left(1-\frac{\varepsilon_k^{\mathrm{F}}-\mu_{\mathrm{F}}}{E_k}\right)}\\ \sqrt{\frac{1}{2}\left(1+\frac{\varepsilon_k^{\mathrm{F}}-\mu_{\mathrm{F}}}{E_k}\right)}\end{pmatrix}, \tag{9}$$

式中$E_k\equiv\sqrt{\left(\varepsilon_k^{\mathrm{F}}-\mu_{\mathrm{F}}\right)^2+|\Delta|^2}$为超导能谱,则Bogoliubov变换矩阵$\hat{R}$为

$$\hat{R}=\begin{pmatrix}\sqrt{\frac{1}{2}\left(1+\frac{\varepsilon_k^{\mathrm{F}}-\mu_{\mathrm{F}}}{E_k}\right)} & \sqrt{\frac{1}{2}\left(1-\frac{\varepsilon_k^{\mathrm{F}}-\mu_{\mathrm{F}}}{E_k}\right)}\\ -\sqrt{\frac{1}{2}\left(1-\frac{\varepsilon_k^{\mathrm{F}}-\mu_{\mathrm{F}}}{E_k}\right)} & \sqrt{\frac{1}{2}\left(1+\frac{\varepsilon_k^{\mathrm{F}}-\mu_{\mathrm{F}}}{E_k}\right)}\end{pmatrix}. \tag{10}$$

为计算方便,定义$u_k\equiv\sqrt{\frac{1}{2}\left(1+\frac{\varepsilon_k^{\mathrm{F}}-\mu_{\mathrm{F}}}{E_k}\right)}$,$v_k\equiv\sqrt{\frac{1}{2}\left(1-\frac{\varepsilon_k^{\mathrm{F}}-\mu_{\mathrm{F}}}{E_k}\right)}$,则变换矩阵$\hat{R}$简化为

$$\hat{R}=\begin{pmatrix}u_k & v_k\\ -v_k & u_k\end{pmatrix}. \tag{11}$$

此时可以得到

$$\begin{aligned}\hat{\beta}_k&=u_k\hat{c}_k-v_k\hat{c}_{-k}^{\dagger},\\ \hat{\beta}_{-k}^{\dagger}&=v_k\hat{c}_k+u_k\hat{c}_{-k}^{\dagger},\end{aligned} \tag{12}$$

且哈密顿量变为

$$\hat{H}_{k,\mathrm{Bo}}^{\mathrm{F}}=\hat{R}^{\dagger}\hat{H}_k^{\mathrm{F}}\hat{R}=\begin{pmatrix}E_k & 0\\ 0 & E_k\end{pmatrix}. \tag{13}$$

由于本节讨论的是费米子体系,准粒子需满足费米子反对易关系,即

$$\begin{aligned}\{\hat{\beta}_k,\hat{\beta}_k^{\dagger}\}_+&=\hat{\beta}_k\hat{\beta}_k^{\dagger}+\hat{\beta}_k^{\dagger}\hat{\beta}_k=\\ &\left(u_k\hat{c}_k-v_k\hat{c}_{-k}^{\dagger}\right)\cdot\left(u_k\hat{c}_k^{\dagger}-v_k\hat{c}_{-k}\right)+\left(u_k\hat{c}_k^{\dagger}-v_k\hat{c}_{-k}\right)\cdot\left(u_k\hat{c}_k-v_k\hat{c}_{-k}^{\dagger}\right)=\\ &u_k{}^2+v_k{}^2=1.\end{aligned} \tag{14}$$

因此可知,对于费米子体系而言,Bogoliubov变换可以通过对体系做幺正变换得到.

## 2　玻色子体系Bogoliubov变换

为了研究玻色子体系如何做Bogoliubov变换从而得到超流,考虑自旋为0、相互作用为接触势能的玻色子体系,其二次量子化之后的哈密顿量为[14]

$$\hat{H}^{\mathrm{B}}=\sum_k\left(\varepsilon_k^{\mathrm{B}}-\mu_{\mathrm{B}}\right)\hat{b}_k^{\dagger}\hat{b}_k+\frac{g_{\mathrm{B}}}{2V}\sum_{k_1,k_2,k_3,k_4}\hat{b}_{k_1}^{\dagger}\hat{b}_{k_2}^{\dagger}\hat{b}_{k_3}\hat{b}_{k_4}, \tag{15}$$

式中$\boldsymbol{k}_1+\boldsymbol{k}_2=\boldsymbol{k}_3+\boldsymbol{k}_4$,$\varepsilon_k^{\mathrm{B}}=\hbar^2|\boldsymbol{k}|^2/\left(2m_{\mathrm{B}}\right)$是玻色子动能,$\mu_{\mathrm{B}}$为化学势,$\hat{b}_k^{\dagger}\left(\hat{b}_k\right)$为玻色子产生(湮灭)算符,式(15)中的第2项为玻色子之间的相互作用项,$g_{\mathrm{B}}$和$V$分别是玻色子之间的相互作用强度和体积.考虑到玻色子在零动量状态下的凝聚情况,即粒子宏观占据零动量态,则有$\left\langle\hat{b}_{k=0}\right\rangle=\sqrt{N_0}$,$N_0\equiv\sqrt{n_0V}$,$N_0$和$n_0$分别为占据在零动量态的粒子数和粒子数密度,当所有粒子凝聚在零动量态时,体系能量对粒子数密度偏导得0,求得化学势为$g_{\mathrm{B}}n_0$,对玻色子

体系做平均场近似,则哈密顿量变为

$$\hat{H}^{\mathrm{B}}_{\mathrm{MF}} = \sum_k \left(\varepsilon^{\mathrm{B}}_k + g_{\mathrm{B}}n_0\right)\hat{b}^{\dagger}_k\hat{b}_k + \frac{g_{\mathrm{B}}n_0}{2}\sum_{k\neq 0}\left(\hat{b}^{\dagger}_k\hat{b}^{\dagger}_{-k} + \hat{b}_k\hat{b}_{-k}\right), \tag{16}$$

在Nambu表象下得

$$\hat{H}^{\mathrm{B}}_{\mathrm{MF}} = \sum_k \hat{\varphi}^{\dagger}_k\hat{H}^{\mathrm{B}}_k\hat{\varphi}_k + \mathrm{const.}, \tag{17}$$

式中$\hat{\varphi}_k$为Nambu表象下的玻色子场算符,有

$$\hat{\varphi}_k = \left(\hat{b}_k, \hat{b}^{\dagger}_{-k}\right)^{\mathrm{T}}, \tag{18}$$

$$\hat{H}^{\mathrm{B}}_k = \frac{1}{2}\begin{pmatrix} \varepsilon^{\mathrm{B}}_k + g_{\mathrm{B}}n_0 & g_{\mathrm{B}}n_0 \\ g_{\mathrm{B}}n_0 & \varepsilon^{\mathrm{B}}_k + g_{\mathrm{B}}n_0 \end{pmatrix}. \tag{19}$$

需要通过做Bogoliubov变换寻找本征表象,并且在本征表象下,哈密顿量矩阵是对角的.首先,按照费米子对角化的方式,尝试玻色子体系是否可以实现Bogoliubov变换,从而对角化玻色子哈密顿量.假设玻色子准粒子算符为$\hat{\alpha}_k$,变换矩阵为$\hat{U}$,则准粒子表象下的算符及哈密顿量与粒子数表象下的算符及哈密顿量的关系为

$$\begin{pmatrix} \hat{\alpha}_k \\ \hat{\alpha}^{\dagger}_{-k} \end{pmatrix} = \hat{U}^{\dagger}\begin{pmatrix} \hat{b}_k \\ \hat{b}^{\dagger}_{-k} \end{pmatrix}, \tag{20}$$

$$\hat{H}^{\mathrm{B}}_{k,\mathrm{Bo}} = \hat{U}^{\dagger}\hat{H}^{\mathrm{B}}_k\hat{U}. \tag{21}$$

解$\hat{H}^{\mathrm{B}}_k$的久期方程

$$\frac{1}{2}\begin{vmatrix} \varepsilon^{\mathrm{B}}_k + g_{\mathrm{B}}n_0 - \lambda & g_{\mathrm{B}}n_0 \\ g_{\mathrm{B}}n_0 & \varepsilon^{\mathrm{B}}_k + g_{\mathrm{B}}n_0 - \lambda \end{vmatrix} = 0, \tag{22}$$

得到2个本征值为

$$\begin{aligned} \lambda_{1,k} &= \frac{1}{2}\left(\varepsilon^{\mathrm{B}}_k + 2g_{\mathrm{B}}n_0\right), \\ \lambda_{2,k} &= \frac{1}{2}\varepsilon^{\mathrm{B}}_k. \end{aligned} \tag{23}$$

本征态为

$$\phi_1(\boldsymbol{k}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \phi_2(\boldsymbol{k}) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}. \tag{24}$$

本征态构成的变化矩阵为

$$\hat{U} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}. \tag{25}$$

则Bogoliubov变换形式为

$$\begin{aligned} \hat{\alpha}_k &= \hat{b}_k + \hat{b}^{\dagger}_{-k}, \\ \hat{\alpha}^{\dagger}_{-k} &= -\hat{b}_k + \hat{b}^{\dagger}_{-k}. \end{aligned} \tag{26}$$

准粒子算符需满足玻色子对易关系,即$\left[\hat{\alpha}_k,\hat{\alpha}^{\dagger}_k\right] = \hat{\alpha}_k\hat{\alpha}^{\dagger}_k - \hat{\alpha}^{\dagger}_k\hat{\alpha}_k = 1$.但通过计算式(26)可知,$\left[\hat{\alpha}_k,\hat{\alpha}^{\dagger}_k\right] = \left[\hat{b}_k,\hat{b}^{\dagger}_k\right] - \left[\hat{b}_{-k},\hat{b}^{\dagger}_{-k}\right] = 1 - 1 = 0$,即用幺正变换来对角化无法满足玻色子对易关系.因此,通过幺正变换对玻色子体系做Bogoliubov变换的方法是不正确的.

通常可以用代数的方法,假设变换矩阵是普适的,在满足玻色子准粒子对易关系的情况下计算出每个矩阵元,但是当考虑高维玻色子体系时,如讨论自旋为1的玻色子时,需要对角化的是6×6的矩阵,若用代数方法对角化哈密顿量存在计算的复杂性.因此,需要找到可行的哈密顿量矩阵对角化方法,来实现Bogoliubov变换.本文构造如何对玻色子体系做矩阵变换来实现Bogoliubov变换.

首先用$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$与$\hat{H}^{\mathrm{B}}_k$相乘得到新的哈密顿量,并定义新哈密顿量

$$\hat{H}' = \begin{pmatrix} \varepsilon^{\mathrm{B}}_k + g_{\mathrm{B}}n_0 & g_{\mathrm{B}}n_0 \\ -g_{\mathrm{B}}n_0 & -\varepsilon^{\mathrm{B}}_k - g_{\mathrm{B}}n_0 \end{pmatrix}, \tag{27}$$

解$\hat{H}'$的久期方程得到超流能谱

$$E_{k,1} = -E_{k,2} = \sqrt{\varepsilon^{\mathrm{B}}_k\left(\varepsilon^{\mathrm{B}}_k + 2g_{\mathrm{B}}n_0\right)}, \tag{28}$$

$E_{k,\alpha}$对应的本征态为$\phi_{\alpha}(\boldsymbol{k}) = \left(\eta_{1\alpha,k} \quad \eta_{2\alpha,k}\right)^{\mathrm{T}}$,$\alpha = 1$、2,代表2个不同的本征态,而本征态的归一化关系为:当$\alpha = 1$时,

$$\left|\eta_{1\alpha,k}\right|^2 - \left|\eta_{2\alpha,k}\right|^2 = 1; \tag{29}$$

当$\alpha = 2$时,

$$\left|\eta_{2\alpha,k}\right|^2 - \left|\eta_{1\alpha,k}\right|^2 = 1. \tag{30}$$

按照此归一化条件计算出本征态为

$$\phi_1(\boldsymbol{k}) = \begin{pmatrix} \sqrt{\frac{1}{2}\left(\frac{\varepsilon^{\mathrm{B}}_k + g_{\mathrm{B}}n_0}{E_{k,1}} + 1\right)} \\ -\sqrt{\frac{1}{2}\left(\frac{\varepsilon^{\mathrm{B}}_k + g_{\mathrm{B}}n_0}{E_{k,1}} - 1\right)} \end{pmatrix}, \tag{31}$$

$$\phi_2(\boldsymbol{k}) = \begin{pmatrix} -\sqrt{\frac{1}{2}\left(\frac{\varepsilon^{\mathrm{B}}_k + g_{\mathrm{B}}n_0}{E_{k,1}} - 1\right)} \\ \sqrt{\frac{1}{2}\left(\frac{\varepsilon^{\mathrm{B}}_k + g_{\mathrm{B}}n_0}{E_{k,1}} + 1\right)} \end{pmatrix}. \tag{32}$$

Bogoliubov变换矩阵$\hat{U}'$的形式为

$$\hat{U}' = \left(\phi_1(\boldsymbol{k}) \quad \phi_2(\boldsymbol{k})\right) = \begin{pmatrix} \sqrt{\frac{1}{2}\left(\frac{\varepsilon^{\mathrm{B}}_k + g_{\mathrm{B}}n_0}{E_{k,1}} + 1\right)} & -\sqrt{\frac{1}{2}\left(\frac{\varepsilon^{\mathrm{B}}_k + g_{\mathrm{B}}n_0}{E_{k,1}} - 1\right)} \\ -\sqrt{\frac{1}{2}\left(\frac{\varepsilon^{\mathrm{B}}_k + g_{\mathrm{B}}n_0}{E_{k,1}} - 1\right)} & \sqrt{\frac{1}{2}\left(\frac{\varepsilon^{\mathrm{B}}_k + g_{\mathrm{B}}n_0}{E_{k,1}} + 1\right)} \end{pmatrix}. \tag{33}$$

为了简化形式,定义 $u_k^{\mathrm{B}} \equiv \sqrt{\frac{1}{2}\left(\frac{\varepsilon_k^{\mathrm{B}} + g_{\mathrm{B}}n_0}{E_{k,1}} + 1\right)}$, $v_k^{\mathrm{B}} \equiv \sqrt{\frac{1}{2}\left(\frac{\varepsilon_k^{\mathrm{B}} + g_{\mathrm{B}}n_0}{E_{k,1}} - 1\right)}$,则

$$\hat{U}' = \begin{pmatrix} u_k^{\mathrm{B}} & -v_k^{\mathrm{B}} \\ -v_k^{\mathrm{B}} & u_k^{\mathrm{B}} \end{pmatrix}. \tag{34}$$

此时准粒子变换形式应为

$$\begin{pmatrix} \hat{\alpha}_k \\ \hat{\alpha}_{-k}^{\dagger} \end{pmatrix} = \hat{U}'^{-1} \begin{pmatrix} \hat{b}_k \\ \hat{b}_{-k}^{\dagger} \end{pmatrix}, \tag{35}$$

对应的代数形式为

$$\begin{aligned} \hat{\alpha}_k &= u_k^{\mathrm{B}}\hat{b}_k + v_k^{\mathrm{B}}\hat{b}_{-k}^{\dagger}, \\ \hat{\alpha}_{-k}^{\dagger} &= v_k^{\mathrm{B}}\hat{b}_k + u_k^{\mathrm{B}}\hat{b}_{-k}^{\dagger}. \end{aligned} \tag{36}$$

准粒子表象下的哈密顿量为

$$\hat{H}'_{\mathrm{Bo}} = \hat{U}'^{-1}\hat{H}'\hat{U}' = \begin{pmatrix} u_k^{\mathrm{B}} & v_k^{\mathrm{B}} \\ v_k^{\mathrm{B}} & u_k^{\mathrm{B}} \end{pmatrix} \begin{pmatrix} \varepsilon_k + g_{\mathrm{B}}n_0 & g_{\mathrm{B}}n_0 \\ -g_{\mathrm{B}}n_0 & -\varepsilon_k - g_{\mathrm{B}}n_0 \end{pmatrix} \begin{pmatrix} u_k^{\mathrm{B}} & -v_k^{\mathrm{B}} \\ -v_k^{\mathrm{B}} & u_k^{\mathrm{B}} \end{pmatrix} = \begin{pmatrix} E_{k,1} & 0 \\ 0 & E_{k,1} \end{pmatrix}. \tag{37}$$

准粒子表象下的算子对易关系满足

$$\left[\hat{\alpha}_k, \hat{\alpha}_k^{\dagger}\right] = \hat{\alpha}_k\hat{\alpha}_k^{\dagger} - \hat{\alpha}_k^{\dagger}\hat{\alpha}_k = \left(u_k^{\mathrm{B}}\hat{b}_k + v_k^{\mathrm{B}}\hat{b}_{-k}^{\dagger}\right)\left(u_k^{\mathrm{B}}\hat{b}_k^{\dagger} + v_k^{\mathrm{B}}\hat{b}_{-k}\right) - \left(u_k^{\mathrm{B}}\hat{b}_k^{\dagger} + v_k^{\mathrm{B}}\hat{b}_{-k}\right)\left(u_k^{\mathrm{B}}\hat{b}_k + v_k^{\mathrm{B}}\hat{b}_{-k}^{\dagger}\right) = u_k^{\mathrm{B}2} - v_k^{\mathrm{B}2} = 1. \tag{38}$$

通过此变换矩阵不仅得到正确的超流能谱,还满足玻色子对易关系,并且对角化体系的哈密顿量.此方法还可以推广到高维体系,只需在 $\hat{H}_k^{\mathrm{B}}$ 的偶数行乘以-1,得到新的哈密顿量 $\hat{H}'$,再解 $\hat{H}'$ 的久期方程,计算本征态时,本征值为正的本征态满足式(29)的归一化关系,本征值为负的本征态满足式(30)的归一化关系.

## 3　结　论

量子力学中幺正变换对角化哈密顿量得到的变换矩阵满足反对易关系,因此,适用于费米子体系.然而玻色子满足对易关系,幺正变换已不再适用.因此,本文研究了玻色子体系哈密顿量的对角化问题,为了满足对易关系,需要将哈密顿量的偶数行乘以-1,再计算久期方程,本征态的归一化条件也因本征值的正负不同而不同,最后将得到的本征态组成变换矩阵,而这个变换矩阵即可完成玻色子体系的Bogoliubov变换.需要说明的是,得到此变换矩阵的方法可以推广到高维体系,从而解决代数方法计算的复杂性.

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# Study on the impurity induced polaron of one-dimensional spin-orbit coupled in spinless Fermi gas

XIE Tianyang, YIN Honghao, WANG Yifan, YANG Mingyue, SUN Qing

(Department of Physics, Capital Normal University, Beijing 100048)

**Abstract:** The basic properties of polaron formed by one-dimensional spin orbit coupling impurities in spinless Fermi gas were studied by using the variational wave function method. The results showed that the momentum of the polaron state was always 0 when the impurity had no spin-orbit coupling, and the polaron state had finite momentum when the impurity had one-dimensional spin-orbit coupling, and it decreased with the increase of the Raman coupling strength. When Zeeman splitting was not zero, the momentum of polaron state would decrease to zero with the increase of Raman coupling strength. The model studied in this paper can be realized under the existing experimental technology, which was helpful to study the few body problem in polarized Fermi gas.

**Keywords:** cold atom; spin-orbit coupling; Fermi gas; polaron

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# Bogoliubov transformation in Boson system

BAI Yixiu, HU Jie

(Department of Physics, Capital Normal University, Beijing 100048)

**Abstract:** The Bogoliubov transformation was a unitary transformation for the fermion system satisfying the anti-commutation relation, which can directly diagonalize the Hamiltonian of the system, while the boson system satisfied the commutative relation and cannot directly diagonalize the matrix. In this paper, the boson system was taken as the research object, and the Hamiltonian was transformed to be diagonalized so that it can be represented in its own representation, and this method can be extended to the high-dimensional spin system.

**Keywords:** cold atom; boson; fermion; Bogoliubov transformation; unitary transformation

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