DOI:10.19789/j.1004-9398.2015.06.003

Hamilton-Jacobi 方程的最大维不变子空间

屈改珠

(渭南师范学院 数学与信息科学学院,陕西 渭南 714000)

摘 要

利用不变子空间方法研究 Hamilton-Jacobi 方程,得到了 Hamilton-Jacobi 方程在它所容许的最大维不变子空间中的完全分类,基于这些不变子空间获得方程的不同形式的广义分离变量解. 文中的结果推广了不变子空间理论在非线性偏微分方程中的应用.

关键词: 不变子空间,Hamilton-Jacobi 方程,广义分离变量解.

中图分类号: O175.29

1 引 言

不变子空间方法是与广义条件对称[1-4]密切相关的构造非线性偏微分方程(组)广义变量分离解的有效方法之一,该方法由 Galaktionov 首次提出,在文献[5]中,Galaktionov 利用不变子空间方法研究带有二次非线性项的演化方程的广义分离变量解. 一般来说,一个偏微分方程所容许的不变子空间包含多项式函数、三角函数、指数函数或者这三种函数混合构成的函数等类型,这些类型的精确解在一些重要的偏微分方程的研究中都具有很重要的意义[6-9].

下面介绍不变子空间方法. 考虑一阶演化方程

$$u_t = F[u], \tag{1}$$

其中 $F[u] \equiv F(x,u,u_x,\cdots,u_{kx})$ 是一个 k 阶非线性微分算子,$F(\cdot)$ 是充分光滑的函数,u_{kx} 表示 u 关于 x 的 k 阶导数,令 n 维线性子空间

$$W_n = L\{f_1(x),\cdots,f_n(x)\} \equiv \{\sum_{i=1}^{n} C_i f_i(x) \mid C_i \in R\},$$

并且 $f_1(x),\cdots,f_n(x)$ 线性无关. 如果算子 F 满足条件 $F[W_n] \subseteq W_n$,则称算子 F 容许不变子空间 W_n. 这就表示方程(1)具有如下形式的广义分离变量解

$$u = \sum_{i=1}^{n} C_i(t) f_i(x),$$

同时 $C_i(t)$ 满足下面的有限维动力系统:

$$\frac{dC_i(t)}{dt} = \psi_i(C_1(t),C_2(t),\cdots,C_n(t)),$$

$$i = 1,2,\cdots,n.$$

注意到 $W_n = L\{f_1(x),\cdots,f_n(x)\}$ 是由 n 阶线性常微分方程的解空间生成

$$L[y] \equiv y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_1(x)y' + a_0(x)y = 0, \tag{2}$$

从(2)可以得到 W_n 关于 F 的不变条件为

$$L[F[u]]\big|_{[H]} \equiv 0, \tag{3}$$

这里用 $[H]$ 表示方程 $L[u] = 0$ 以及它关于 x 求各阶导数后的等式.

2 主要结果

本文主要利用不变子空间方法研究具有一般形式的 Hamilton-Jacobi 方程

$$u_t = g(u,u_x) \tag{4}$$

的最大维不变子空间,这里 $g(u,u_x)$ 是关于参量 u,u_x 的光滑函数.

根据最大维定理,我们讨论方程(4)容许的最大维(三维)不变子空间 W_3,且 W_3 由如下线性常微分方程定义:

$$L[y] = y''' + a_2y'' + a_1y' + a_0y = 0, \tag{5}$$

则相应的不变条件变为

收稿日期:2014-10-30

* 国家自然科学基金资助项目(11371293);陕西省教育厅基金项目(14JK1246);渭南市基础研究计划项目(2013JCYJ-4);渭南师范学院理工类科研项目(15YKS005);渭南师范学院校级特色学科建设项目(14TSXK02).

(C)1994-2020 China Academic Journal Electronic Publishing House. All rights reserved. http://www.cnki.net

$$(D^3g + a_2D^2g + a_1Dg + a_0g)\big|_{[4]} = 0, \quad (6)$$

由(6)可确定方程(4)中未知函数的表达式,并再次代入(6),得到一个代数方程组,借助符号计算软件 Maple 求解该代数方程组,即可得到下面的命题.

命题 1 若由方程(5)所确定的三维不变子空间在(4)下保持不变,则存在以下五种分类结果:

(i) $u_t = c_1u_x^2 + c_4u_x + c_7u + c_8, W_3 = \{1, x, x^2\}$, $(y''' = 0)$;

(ii) $u_t = c_1u_x^2 + c_4u_x + c_1u^2 + c_7u + c_8, W_3 = \{1, \sin x, \cos x\}$, $(y''' + y' = 0)$;

(iii) $u_t = c_1u_x^2 + c_4u_x - c_1u^2 + c_7u + c_8, W_3 = \{1, e^x, e^{-x}\}$, $(y''' - y' = 0)$;

(iv) $u_t = c_1u_x^2 + \frac{4}{3}c_1uu_x + c_4u_x + \frac{4}{9}c_1u^2 + c_7u + c_8$,

$W_3 = \{1, e^{-\frac{1}{3}x}, e^{-\frac{2}{3}x}\}$, $\left(y''' + y'' + \frac{2}{9}y' = 0\right)$;

(v) $u_t = c_1u_x^2 - \frac{4}{3}c_1uu_x + c_4u_x + \frac{4}{9}c_1u^2 + c_7u + c_8$,

$W_3 = \{1, e^{\frac{1}{3}x}, e^{\frac{2}{3}x}\}$, $\left(y''' - y'' + \frac{2}{9}y' = 0\right)$.

证明过程类似于文献[7]. 下面给出一个求相应偏微分方程精确解的例子.

例 1 考虑方程 $u_t = c_1u_x^2 + c_4u_x + c_7u + c_8$, 它容许三维多项式不变子空间

$W_3 = \{1, x, x^2\}$, 即方程的解为

$$u(x,t) = C_1(t) + C_2(t)x + C_3(t)x^2,$$

其中 $C_i(t)$ 满足下面的常微分方程组

$$C'_1(t) = c_1(C_2(t))^2 + c_7C_1(t) + c_4C_2(t) + c_8,$$
$$C'_2(t) = c_7C_2(t) + 4c_1C_2(t)C_3(t) + 2c_4C_3(t),$$
$$C'_3(t) = 4c_1(C_3(t))^2 + c_7C_3(t).$$

求解上述方程组,可得原方程具有以下形式的解

$$u = b_1e^{c_1t} + \frac{1}{c_7b_3e^{-c_7t} - 4c_1} \cdot \left[\frac{16c_7c_8 + b_2^2 + 4c_4^2c_7^2t^2 - 4b_3c_7c_8e^{-c_7t}}{4c_7} + c_4(b_2 + 2c_7x)t + b_2x + c_7x^2\right],$$

这里 $b_i(i = 1, 2, 3)$ 是任意实数.

参 考 文 献

[1] FOKAS A S, Liu Q M. Generalized conditional symmetries and exact solutions of nonintegrable equations [J]. Theor. Math. Phys, 1994, 99: 263 – 277.

[2] FOKAS A S, Liu Q M. Nonlinear interaction of traveling waves of nonintegrable equations [J]. Phys. Rev. Lett, 1994, 72: 3293 – 3296.

[3] ZHDANOV R Z. Conditional Lie-Backlund symmetry and reduction of evolution equations [J]. J. Phys. A: Math. Gen, 1995, 28: 3841 – 3850.

[4] Qu C Z. Group classification and generalized conditional symmetry reduction of the nonlinear diffusion convection equation with a nonlinear source [J]. Stud. Appl. Math, 1997, 99: 107 – 136.

[5] GALAKTIONOV V A. Invariant subspaces and new explicit solutions to evolution equations with quadratic nonlinearities [J]. Proc. Royal. Soc. Edinburgh, 1995, 125: 225 – 246.

[6] SVIRSHCHEVSKII S R. Symmetries of linear ODEs and generalized separation of variables in nonlinear equations [J]. Phys. Lett. A, 1995, 199: 344 – 348.

[7] GALAKTIONOV V A, SVIRSHCHEVSKII S R. Exact Solutions and Invariant Subspaces of Nonlinear Partial Differential Equations in Mechanics and Physics [M]. Chapman and Hall/CRC, London, 2007.

[8] Qu C Z, Zhu C R. Classification of coupled systems with two-component nonlinear diffusion equations by the invariant subspace method [J]. J. Phys. A: Math. Theor, 2009, 42: 475201 – 475227.

[9] Ma W X. A refined invariant subspace method and applications to evolution equations [J]. Sci. China. Math, 2012, 55: 1769 – 1778.

(下转第 23 页)

(C)1994-2020 China Academic Journal Electronic Publishing House. All rights reserved. http://www.cnki.net

The Analysis of Wechat Users´ Relationships Network Based on the Block Model

Yi Jianhong　Wang Huan

(School of Public Management, Beijing University of Posts and Telecommunications　100876)

Abstract

The mature development of Wechat makes it became an important platform for people to maintain interpersonal relationship, and it plays a very important role in interpersonal communication and dissemination of information. The paper chooses a Wechat user and its all friends as the object of study based on the the research about Wechat and quantitative analysis to format a users´ relationship network; at the same time, it used the block model to analyze the relationship between the groups and the overall structure Wechat users´ relationships network. Based on the research , it finds that there are a large number of subgroups in the users´ relationship networks, and some subgroups have shareing members; internal between subgroups, and each subgroup has interaction and communication; the distribution of subgroups showed the pattern of difference sequence, and the relationship internal the groups is stronger than between groups, and strong ties predominate, so weak ties are not obvious.

Key words: block model, Wechat, users´ relationships network, the differential mode of association.

(上接第 12 页)

Maximal Dimension Invariant Subspaces of Hamilton-Jacobi Equations

Qu Gaizhu

(School of Mathematics and Information Science, Weinan Normal University, Weinan Shannxi　714000)

Abstract

Using the invariant subspace method, Hamilton-Jacobi equations are discussed, the full classifications of maximal dimension invariant subspace admitted by the Hamilton-Jacobi equations are derived, moreover, several different types of separation of variables solutions are obtained. The obtained results further extend the applications of invariant subspace theory in the PDEs.

Key words: invariant subspace, Hamilton-Jacobi equations, generalized separation of variables solution

作者简介　屈改珠(1978 -),女,陕西蒲城人,西北大学数学学院在读博士,主要从事偏微分方程研究.

(C)1994-2020 China Academic Journal Electronic Publishing House. All rights reserved.　http://www.cnki.net