# 形如$k\varphi(n)=\varphi_2(n)+S(n^m)$的两个方程的可解性*

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**摘要**:讨论包含Euler函数$\varphi(n)$、广义Euler函数$\varphi_2(n)$与Smarandache函数$S(n)$的2个方程的可解性,基于Euler函数$\varphi(n)$,广义Euler函数$\varphi_2(n)$与Smarandache函数$S(n)$的性质及其各自的计算公式,利用初等的方法与Guass函数[$n$]的性质,得到方程$3\varphi(n)=\varphi_2(n)+S(n^{30})$无正整数解,以及方程$2\varphi(n)=\varphi_2(n)+S(n^{28})$仅有正整数解$n=288, 1083, 1444, 2166$.

**关键词**:Euler函数$\varphi(n)$;广义Euler函数$\varphi_2(n)$;Smarandache函数$S(n)$;正整数解

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## 0 引 言

数论函数方程的可解性问题的研究是数论中的一个热点研究内容.令$\varphi(n)$为Euler函数,其是数论中一个重要的数论函数,包含数论函数$\varphi(n)$方程的可解性有着众多的研究内容,如文献[1-3].令$\varphi_e(n)$为广义Euler函数,是由蔡天新[4]在研究将Lehmer同余式从模素数的平方推广到模任意整数的平方时,所提出的一个数论函数.对于包含数论函数$\varphi_e(n)$方程的可解性有着丰富的研究成果,如文献[5-7].令$S(n)$为Smarandache函数,其定义为$S(n)=\min\{m: n\in Z^+, n|m!\}$,是由美籍罗马尼亚数论专家Florentin Smarandache于1993年在其撰写的*Only Problems, Not Solutions*一书中所提出的一个数论函数.对于包含数论函数$S(n)$方程的可解性也有着颇多的研究成果,如文献[8-10].

对于包含Euler函数$\varphi(n)$、广义Euler函数$\varphi_2(n)$与Smarandache函数$S(n)$方程的可解性问题,文献[11]讨论了方程$2\varphi(n)=\varphi_2(n)+S(n^{25})$的可解性,并给出了其有3个正整数解.本文将基于Euler函数$\varphi(n)$、广义Euler函数$\varphi_2(n)$与Smarandache函数$S(n)$的性质及其各自的计算公式,讨论方程

$$3\varphi(n)=\varphi_2(n)+S(n^{30}) \tag{1}$$

和方程

$$2\varphi(n)=\varphi_2(n)+S(n^{28}) \tag{2}$$

的可解性,利用初等的方法与Guass函数[$n$]的性质给出正整数解情况.

## 1 几个基本的引理

**引理1**[12] 如果$n=\prod_{i=1}^{k}q_i^{\alpha_i}$,则$S(n)=\max\{S(q_1^{\alpha_1}), S(q_2^{\alpha_2}), \cdots, S(q_k^{\alpha_k})\}$.

**引理2**[12] 对于素数$p$和正整数$k$,有$S(p^k)\leqslant kp$;特别地,当$k<p$时,有$S(p^k)=kp$.

**引理3**[13] 当$n\geqslant 3$时,有$\varphi(n)$为偶数.

**引理4**[14] 当$n\geqslant 3$时,有$\varphi_2(n)=2^{-1}\varphi(n)$.

## 2 定理及其证明

**定理1** 方程(1)$3\varphi(n)=\varphi_2(n)+S(n^{30})$无正整数解.

**证明** 显然$n=1,2$不是方程(1)的正整数解,设$n=\prod_{i=1}^{k}q_i^{\alpha_i}\geqslant 3$是正整数$n$的标准分解式,其中$q_1<q_2<\cdots<q_k$是质数,由引理1可得

$$S(n)=\max\{S(q_1^{\alpha_1}), S(q_2^{\alpha_2}), \cdots, S(q_k^{\alpha_k})\}=S(q^{\alpha}), \tag{3}$$

式中$n=q^{\alpha}n_1$,$q$是$n$的质因数,$\alpha$是$q$在$n$中的指数,$(q,n_1)=1$,则有

$$\varphi(n)=\varphi(q^{\alpha}n_1)=\varphi(q^{\alpha})\varphi(n_1)=q^{\alpha-1}(q-1)\varphi(n_1), \tag{4}$$

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结合引理4与方程(1),进一步有

$$5q^{\alpha-1}(q-1)\varphi(n_1)=2S(n^{30})=2S(q^{30\alpha}). \quad (5)$$

**情况1**　当$\alpha=1$

当$q=2$时,由式(5)有$5\varphi(n_1)=2S(2^{30})=64$无解;当$q=3$时,由式(5)有$10\varphi(n_1)=2S(3^{30})=2\times 63$无解;当$q=5$时,由式(5)有$20\varphi(n_1)=2S(5^{30})=2\times 125$无解;当$q=7$时,由式(5)有$30\varphi(n_1)=2S(7^{30})=2\times 189$无解;当$q=11$时,由式(5)有$50\varphi(n_1)=2S(11^{30})=2\times 308$无解;当$q=13$时,由式(5)有$60\varphi(n_1)=2S(13^{30})=2\times 364$无解;当$q=17$时,由式(5)有$80\varphi(n_1)=2S(17^{30})=2\times 493$无解;当$q=19$时,由式(5)有$90\varphi(n_1)=2S(19^{30})=2\times 551$无解;当$q=23$时,由式(5)有$110\varphi(n_1)=2S(23^{30})=2\times 667$无解;当$q=29$时,由式(5)有$140\varphi(n_1)=2S(29^{30})=2\times 841$无解;当$q\geqslant 31$时,由引理2,则由式(5)有$5(q-1)\varphi(n_1)=2S(q^{30})=60q$,即$(q-1)\varphi(n_1)=12q$,从而有$(q-1)\mid 12q$,而$(q-1,q)=1$,则$(q-1)\mid 12$,这不可能.

**情况2**　当$\alpha=2$

当$q=2$时,由式(5)有$10\varphi(n_1)=2S(2^{60})=2\times 64$无解;当$q=3$时,由式(5)有$30\varphi(n_1)=2S(3^{60})=2\times 126$无解;当$q=5$时,由式(5)有$100\varphi(n_1)=2S(5^{60})=2\times 250$,从而有$\varphi(n_1)=5$,由引理3可知$\varphi(n_1)=5$无解;当$q=7$时,由式(5)有$210\varphi(n_1)=2S(7^{60})=2\times 364$无解;当$q=11$时,由式(5)有$550\varphi(n_1)=2S(11^{60})=2\times 605$无解;当$q=13$时,由式(5)有$780\varphi(n_1)=2S(13^{60})=2\times 728$无解;当$q=17$时,由式(5)有$1\,360\varphi(n_1)=2S(17^{60})=2\times 969$无解;当$q=19$时,由式(5)有$1\,710\varphi(n_1)=2S(19^{60})=2\times 1\,083$无解;当$q=23$时,由式(5)有$2\,530\varphi(n_1)=2S(23^{60})=2\times 1\,334$无解;当$q\geqslant 29$时,由式(5)有$5q(q-1)\varphi(n_1)=2S(q^{60})$. 由引理2有$2S(q^{60})\leqslant 2\times 60q=120q$. 而当$q\geqslant 29$时,有$5q(q-1)\geqslant 140q>120q$,得出矛盾.

**情况3**　当$\alpha=3$

当$q=2$时,由式(5)有$20\varphi(n_1)=2S(2^{90})=2\times 96$无解;当$q=3$时,由式(5)有$90\varphi(n_1)=2S(3^{90})=2\times 186$无解;当$q=5$时,由式(5)有$500\varphi(n_1)=2S(5^{90})=2\times 370$无解;当$q\geqslant 7$时,由式(5)有$5q^2(q-1)\varphi(n_1)=2S(q^{90})$. 由引理2有$2S(q^{90})\leqslant 2\times 90q=180q$. 而当$q\geqslant 7$时,有$5q^2(q-1)\geqslant 210q>180q$,得出矛盾.

**情况4**　当$\alpha=4$

当$q=2$时,由式(5)有$40\varphi(n_1)=2S(2^{120})=2\times 126$无解;当$q=3$时,由式(5)有$270\varphi(n_1)=2S(3^{120})=2\times 243$无解;当$q\geqslant 5$时,由式(5)有$5q^3(q-1)\varphi(n_1)=2S(q^{120})$. 由引理2有$2S(q^{120})\leqslant 2\times 120q=240q$. 而当$q\geqslant 5$时,有$5q^3(q-1)\geqslant 500q>240q$,得出矛盾.

**情况5**　当$\alpha=5$

当$q=2$时,由式(5)有$80\varphi(n_1)=2S(2^{150})=2\times 154$无解;当$q=3$时,由式(5)有$810\varphi(n_1)=2S(3^{150})=2\times 306$无解;当$q\geqslant 5$时,由式(5)有$5q^4(q-1)\varphi(n_1)=2S(q^{150})$. 由引理2有$2S(q^{150})\leqslant 2\times 150q=300q$. 而当$q\geqslant 5$时,有$5q^4(q-1)\geqslant 2\,500q>300q$,得出矛盾.

**情况6**　当$\alpha=6$

当$q=2$时,由式(5)有$160\varphi(n_1)=2S(2^{180})=2\times 184$无解;当$q\geqslant 3$时,由式(5)有$5q^5(q-1)\varphi(n_1)=2S(q^{180})$. 由引理2有$2S(q^{180})\leqslant 2\times 180q=360q$. 而当$q\geqslant 3$时,有$5q^5(q-1)\geqslant 810q>360q$,得出矛盾.

**情况7**　当$\alpha=7$

当$q=2$时,由式(5)有$320\varphi(n_1)=2S(2^{210})=2\times 216$无解;当$q\geqslant 3$时,由式(5)有$5q^6(q-1)\varphi(n_1)=2S(q^{210})$. 由引理2有$2S(q^{210})\leqslant 2\times 210q=420q$. 而当$q\geqslant 3$时,有$5q^6(q-1)\geqslant 2\,430q>420q$,得出矛盾.

**情况8**　当$\alpha=8$

当$q=2$时,由式(5)有$640\varphi(n_1)=2S(2^{240})=2\times 248$无解;当$q\geqslant 3$时,由式(5)有$5q^7(q-1)\varphi(n_1)=2S(q^{240})$. 由引理2有$2S(q^{240})\leqslant 2\times 240q=480q$. 而当$q\geqslant 3$时,有$5q^7(q-1)\geqslant 7\,290q>480q$,得出矛盾.

**情况9**　当$\alpha=9$

当$q\geqslant 2$时,由式(5)有$5q^8(q-1)\varphi(n_1)=2S(2^{270})$. 由引理2有$2S(q^{270})\leqslant 2\times 270q=540q$. 而当$q\geqslant 2$时,有$5q^8(q-1)\geqslant 640q>540q$,得出矛盾.

**情况10**　当$\alpha\geqslant 10$

当$q=2$时,由式(5)有$5q^{\alpha-1}\varphi(n_1)=2S(2^{30\alpha})$. 由引理2有$2S(q^{30\alpha})\leqslant 120\alpha$. 而当$\alpha\geqslant 10$时,有$5\times 2^{\alpha-1}\varphi(n_1)\geqslant 5\times 2^{\alpha-1}>5\times 2^5\times 2^{\alpha-6}=160\times 2^{\alpha-6}>160\alpha$,得

出矛盾;当$q \geqslant 3$时,由式(5)有$5q^{\alpha-1}(q-1)\varphi(n_1) = 2S(q^{30\alpha})$. 由引理2有$2S(q^{30\alpha}) \leqslant 2 \times 30\alpha q = 60\alpha q$,则$5q^{\alpha-2}(q-1)\varphi(n_1) \leqslant 60\alpha$. 而当$q \geqslant 3$时,有$5q^{\alpha-2}(q-1)\varphi(n_1) \geqslant 5 \times 2 \times 3^{\alpha-2} = 5 \times 2 \times 3^2 \times 3^{\alpha-4} > 90\alpha$,得出矛盾.

综合以上情况的讨论,可得定理1的结论.

**定理2** 方程(2)$2\varphi(n)=\varphi_2(n)+S(n^{28})$仅有正整数解$n = 288, 1083, 1444, 2166$.

**证明** 显然$n = 1, 2$不是方程(2)的正整数解,设$n = \prod_{i=1}^{k} q_i^{\alpha_i} \geqslant 3$是正整数$n$的标准分解式,其中$q_1 < q_2 < \cdots < q_k$是质数,由引理1可得

$$S(n) = \max\{S(q_1^{\alpha_1}), S(q_2^{\alpha_2}), \cdots, S(q_k^{\alpha_k})\} = S(q^{\alpha}), \tag{6}$$

式中$q$是正整数$n$的质因数,$\alpha$是$q$在正整数$n$标准分解中的指数,且$n = q^{\alpha}n_1, (q, n_1) = 1$,则有

$$\varphi(n) = \varphi(q^{\alpha}n_1) = \varphi(q^{\alpha})\varphi(n_1) = q^{\alpha-1}(q-1)\varphi(n_1). \tag{7}$$

由引理4,结合式(6)与式(7),则方程(2)可转化为

$$3q^{\alpha-1}(q-1)\varphi(n_1) = 2S(n^{28}) = 2S(q^{28\alpha}). \tag{8}$$

**情况1** 当$\alpha = 1$

当$q = 2$时,由式(8)有$3\varphi(n_1) = 2S(2^{28}) = 2 \times 32$无解. 当$q = 3$时,由式(8)有$6\varphi(n_1) = 2S(3^{28}) = 2 \times 60$,则有$\varphi(n_1) = 20$,结合$(q, n_1) = 1$,则$n_1 = 25, 44, 50$,因而$n = 75, 132, 150$. 当$n = 75$时,则$n^{28} = 3^{28} \times 5^{56}$,而$S(3^{28}) = 60, S(5^{56}) = 230$,则$n = 75$不是方程(2)的解;同理可得$n = 132, 150$也不是方程(2)的解. 当$q = 5$时,由式(8)有$12\varphi(n_1) = 2S(5^{28}) = 2 \times 120$,则有$\varphi(n_1) = 20$,结合$(q, n_1) = 1$,则$n_1 = 33, 44, 66$,因而$n = 165, 220, 330$. 经检验,$n = 165, 220, 330$不是方程(2)的解. 当$q = 7$时,由式(8)有$18\varphi(n_1) = 2S(7^{28}) = 2 \times 175$无解. 当$q = 11$时,由式(8)有$30\varphi(n_1) = 2S(11^{28}) = 2 \times 286$无解. 当$q = 13$时,由式(8)有$36\varphi(n_1) = 2S(13^{28}) = 2 \times 338$无解. 当$q = 17$时,由式(8)有$48\varphi(n_1) = 2S(17^{28}) = 2 \times 459$无解. 当$q = 19$时,由式(8)有$54\varphi(n_1) = 2S(19^{28}) = 2 \times 513$,则有$\varphi(n_1) = 19$,由引理3可知,$\varphi(n_1) = 19$无解. 当$q = 23$时,由式(8)有$66\varphi(n_1) = 2S(23^{28}) = 2 \times 621$无解. 当$q = 29$时,由式(8)有$84\varphi(n_1) = 2S(29^{28}) = 2 \times 812$无解. 当$q \geqslant 31$时,由引理2,则由式(8)有$3(q-1)\varphi(n_1) = 2S(q^{28}) = 56q$,因$3 \nmid 56$且$3 \nmid q$,因而$3(q-1)\varphi(n_1) = 2S(q^{28}) = 56q$无解.

**情况2** 当$\alpha = 2$

当$q = 2$时,由式(8)有$6\varphi(n_1) = 2S(2^{56}) = 2 \times 60$,则有$\varphi(n_1) = 20$,结合$(q, n_1) = 1$,则$n_1 = 25, 33$,因而$n = 100, 132$. 经检验,$n = 100, 132$不是方程(2)的解. 当$q = 3$时,由式(8)有$18\varphi(n_1) = 2S(3^{56}) = 2 \times 117$无解;当$q = 5$时,由式(8)有$60\varphi(n_1) = 2S(5^{56}) = 2 \times 230$无解;当$q = 7$时,由式(8)有$126\varphi(n_1) = 2S(7^{56}) = 2 \times 343$无解;当$q = 11$时,由式(8)有$330\varphi(n_1) = 2S(11^{56}) = 2 \times 572$无解;当$q = 13$时,由式(8)有$468\varphi(n_1) = 2S(13^{56}) = 2 \times 676$无解;当$q = 17$时,由式(8)有$816\varphi(n_1) = 2S(17^{56}) = 2 \times 901$无解;当$q = 19$时,由式(8)有$1026\varphi(n_1) = 2S(19^{56}) = 2 \times 1026$,则有$\varphi(n_1) = 2$,结合$(q, n_1) = 1$,则$n_1 = 3, 4, 6$,因而$n = 3 \times 19^2 = 1083$,$n = 4 \times 19^2 = 1444$,$n = 6 \times 19^2 = 2166$,经检验,$n = 1083, 1444, 2166$是方程(2)的解;当$q = 23$时,由式(8)有$1518\varphi(n_1) = 2S(23^{56}) = 2 \times 1242$无解;当$q = 29$时,由式(8)有$2436\varphi(n_1) = 2S(29^{56}) = 2 \times 1595$无解;当q = 31时,由式(8)有$2790\varphi(n_1) = 2S(31^{56}) = 2 \times 1705$无解;当$q = 37$时,由式(8)有$3396\varphi(n_1) = 2S(37^{56}) = 2 \times 2035$无解;当$q \geqslant 41$时,由式(8)有$3q(q-1)\varphi(n_1) = 2S(q^{56})$. 由引理2有$2S(q^{56}) \leqslant 2 \times 56q = 112q$. 而当$q \geqslant 41$时,有$3q(q-1) \geqslant 120q > 112q$,得出矛盾.

**情况3** 当$\alpha = 3$

当$q = 2$时,由式(8)有$12\varphi(n_1) = 2S(2^{84}) = 2 \times 88$无解;当$q = 3$时,由式(8)有$54\varphi(n_1) = 2S(3^{84}) = 2 \times 171$无解;当$q = 5$时,由式(8)有$300\varphi(n_1) = 2S(5^{84}) = 2 \times 345$无解;当$q = 7$时,由式(8)有$882\varphi(n_1) = 2S(7^{84}) = 2 \times 511$无解;当$q \geqslant 11$时,由式(8)有$3q^2(q-1)\varphi(n_1) = 2S(q^{84})$. 由引理2有$2S(q^{84}) \leqslant 2 \times 84q = 168q$. 而当$q \geqslant 11$时,有$3q^2(q-1) \geqslant 330q$,得出矛盾.

**情况4** 当$\alpha = 4$

当$q = 2$时,由式(8)有$24\varphi(n_1) = 2S(2^{112}) = 2 \times 116$无解;当$q = 3$时,由式(8)有$162\varphi(n_1) = 2S(3^{112}) = 2 \times 231$无解;当$q \geqslant 5$时,由式(8)有$3q^3(q-1)\varphi(n_1) = 2S(q^{112})$. 由引理2有$2S(q^{112}) \leqslant 2 \times 112q = 224q$. 而当$q \geqslant 5$时,有$3q^3(q-1) \geqslant 300q$,得出矛盾.

**情况5**　当$\alpha=5$

当$q=2$时,由式(8)有$48\varphi(n_1)=2S(2^{140})=2\times144$,则有$\varphi(n_1)=6$,结合$(q,n_1)=1$,则$n_1=7,9$,因而$n=2^5\times7=224$,$n=2^5\times3^2=288$.经检验,此时$n=224$不是方程(2)的解,$n=288$是方程(2)的解;当$q=3$时,由式(8)有$486\varphi(n_1)=2S(3^{140})=2\times285$无解;当$q\geqslant5$时,由式(8)有$3q^4(q-1)\varphi(n_1)=2S(q^{140})$.由引理2有$2S(q^{140})\leqslant2\times140q=280q$.而当$q\geqslant5$时,有$3q^4(q-1)\geqslant1500q>280q$,得出矛盾.

**情况6**　当$\alpha=6$

当$q=2$时,由式(8)有$96\varphi(n_1)=2S(2^{168})=2\times172$无解;当$q\geqslant3$时,由式(8)有$3q^5(q-1)\varphi(n_1)=2S(q^{168})$.由引理2有$2S(q^{168})\leqslant2\times168q=336q$.而当$q\geqslant3$时,有$3q^5(q-1)\geqslant486q$,得出矛盾.

**情况7**　当$\alpha=7$

当$q=2$时,由式(8)有$192\varphi(n_1)=2S(2^{196})=2\times200$无解;当$q\geqslant3$时,由式(8)有$3q^6(q-1)\varphi(n_1)=2S(q^{196})$.由引理2有$2S(q^{196})\leqslant2\times196q=392q$.而当$q\geqslant3$时,有$3q^6(q-1)\geqslant1458q>392q$,得出矛盾.

**情况8**　当$\alpha=8$

当$q=2$时,由式(8)有$384\varphi(n_1)=2S(2^{224})=2\times228$无解;当$q\geqslant3$时,由式(8)有$3q^7(q-1)\varphi(n_1)=2S(q^{224})$.由引理2有$2S(q^{224})\leqslant2\times224q=448q$.而当$q\geqslant3$时,有$3q^7(q-1)\geqslant4374q$,得出矛盾.

**情况9**　当$\alpha=9$

当$q=2$时,由式(8)有$768\varphi(n_1)=2S(2^{252})=2\times256$无解;当$q\geqslant3$时,由式(8)有$3q^8(q-1)\varphi(n_1)=2S(q^{252})$.由引理2有$2S(q^{252})\leqslant504q$.而当$q\geqslant3$时,有$3q^8(q-1)\geqslant13122q>504q$,得出矛盾.

**情况10**　当$\alpha=10$

当$q=2$时,由式(8)有$1536\varphi(n_1)=2S(2^{280})=2\times284$无解;当$q\geqslant3$时,由式(8)有$3q^9(q-1)\varphi(n_1)=2S(q^{280})$.由引理2有$2S(q^{280})\leqslant560q$.而当$q\geqslant3$时,有$3q^9(q-1)\geqslant39366q$,得出矛盾.

**情况11**　当$\alpha\geqslant11$

由式(8)有$3q^{\alpha-1}(q-1)\varphi(n_1)=2S(2^{28\alpha})$.由引理2有$2S(q^{28\alpha})\leqslant56\alpha q$,则$3q^{\alpha-2}(q-1)\varphi(n_1)\leqslant56\alpha$.而当$q\geqslant2$时,有$3q^{\alpha-2}(q-1)\varphi(n_1)\geqslant3\times2^{\alpha-2}=3\times2^5\times2^{\alpha-7}>96\alpha$,得出矛盾.

综合以上情况的讨论,可得定理2的结论.

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# Teaching practice of flipped classroom for the course of Engineering Graphics:taking "drawing method of combinations" for example

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**Abstract:** In order to explore a new teaching model and alleviate the contradiction between heavy teaching content and less class hours in "Engineering Graphics", the teaching model of "Flipping Classroom" was adopted, task exercises were chosen and classroom discussion units were designed according to the basic teaching requirements and teaching objectives. Through students' representation for task exercises and student-student communication and student-teacher exchanging under the guidance of teacher in classroom activities, students can absorb knowledge internally and complete the construction of knowledge system in limited teaching hours. Practice showed that the "Flipping Classroom" can not only save the teaching hours, but also increase the ratio of students with good and moderate grades.

**Keywords:** Engineering Graphics; flipping classroom; classroom units; teaching hours

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# Solvabilities of two equations of the form $k\varphi(n)=\varphi_2(n)+S(n^m)$

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**Abstract:** The solvabilities of two equations involving the Euler function $\varphi(n)$, the Generalized Euler function $\varphi_2(n)$ and the Smarandache function $S(n)$ were discussed. Based on the properties of Euler function $\varphi(n)$, generalized Euler function $\varphi_2(n)$ and Smarandache function $S(n)$ and their respective calculation formulas, we obtained the equation $3\varphi(n)=\varphi_2(n)+S(n^{30})$ has no positive integer solution, and the equation $2\varphi(n)=\varphi_2(n)+S(n^{28})$ has only positive integer solutions $n$=288, 1 083, 1 444, 2 166 by using the elementary method and the properties of Guass function $[n]$.

**Keywords:** Euler function $\varphi(n)$; generalized Euler function $\varphi_2(n)$; Smarandache function $S(n)$; positive integer solution

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