分形集上的加权 Ostrowski 型双边不等式

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摘要:基于局部分数阶微积分理论,针对具有有界的一阶局部分数阶导数的函数,通过建立恒等式,用引入参数求最值的方法,得到分形集上的加权 Ostrowski 型双边不等式.

关键词: Ostrowski 型不等式;局部分数阶导数;局部分数阶积分;分形集;权函数

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0 引 言

设 $f:[a,b]\to\mathbf{R}$ 在 $[a,b]$ 上连续,在 (a,b) 上可微,且对任意 $x\in(a,b)$ 有 $|f'(x)|\leqslant M$,则有

$$\left|f(x)-\frac{1}{b-a}\int_a^b f(t)\,\mathrm{d}t\right|\leqslant \left(\frac{1}{4}+\frac{\left(x-\frac{a+b}{2}\right)^2}{(b-a)^2}\right)(b-a)M, \tag{1}$$

式(1)称为 Ostrowski 不等式[1]. 有关 Ostrowski 不等式的改进和推广见文献[2-6].

下面使用 Gao-Yang-Kang 的方法来描述局部分数阶导数和积分.

定义 1[7] 设 $f:\mathbf{R}\to\mathbf{R}^\alpha$ 是不可微函数,如果对任意 $\varepsilon>0$,存在 $\delta>0$,使得当 $|x-x_0|<\delta$ 时,有 $|f(x)-f(x_0)|<\varepsilon^\alpha$,则称 f 在点 x_0 处局部分数阶连续. 若 f 在区间 $I\subseteq\mathbf{R}$ 上局部分数阶连续,则记为 $f\in C_\alpha(I)$.

定义 2[7] 设 $f\in C_\alpha(a,b)$,则 f 在点 x_0 处局部分数阶导数定义为

$$f^{(\alpha)}(x_0)=\frac{\mathrm{d}^\alpha f(x)}{\mathrm{d}x^\alpha}\Big|_{x=x_0}=\lim_{x\to x_0}\frac{\Gamma(1+\alpha)(f(x)-f(x_0))}{(x-x_0)^\alpha}.$$

若对任意 $x\in I\subseteq\mathbf{R}$ 时存在 $f^{(\alpha)}(x)$,则记为 $f\in D_\alpha(I)$.

定义 3[7] 设 $f\in C_\alpha[a,b]$,则 f 在区间 $[a,b]$ 上的 α 阶局部分数阶积分定义为

$${}_aI_b^{(\alpha)}f(x)=\frac{1}{\Gamma(1+\alpha)}\int_a^b f(t)(\mathrm{d}t)^\alpha=\frac{1}{\Gamma(1+\alpha)}\lim_{\Delta t\to 0}\sum_{j=0}^{N-1}f(t_j)(\Delta t_j)^\alpha,$$

其中,$a=t_0<t_1<\cdots<t_{N-1}<t_N=b$,$\Delta t_j=t_{j+1}-t_j(j=0,1,\cdots,N-1)$,$\Delta t=\max\{\Delta t_1,\Delta t_2,\cdots,\Delta t_{N-1}\}$.

规定当 $a=b$ 时,${}_aI_b^{(\alpha)}f(x)=0$;当 $a>b$ 时,${}_aI_b^{(\alpha)}f(x)=-{}_bI_a^{(\alpha)}f(x)$. 若对任意 $x\in[a,b]$ 存在 ${}_aI_x^{(\alpha)}f(t)$,则记为 $f\in I_x^{(\alpha)}[a,b]$.

设区间 $I\subseteq\mathbf{R}$,I° 是 I 的内部,$f:I^\circ\to\mathbf{R}^\alpha$,$f\in D_\alpha(I^\circ)$,$a,b\in I^\circ$,$a<b$,$f^{(2\alpha)}\in C_\alpha[a,b]$,则对任意 $x\in[a,b]$ 有

$$\left|f(x)-\frac{\Gamma(1+\alpha)}{(b-a)^\alpha}{}_aI_b^{(\alpha)}f(t)\right|\leqslant \frac{2^\alpha\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}\left[\frac{1}{4^\alpha}+\left(\frac{x-\frac{a+b}{2}}{b-a}\right)^{2\alpha}\right](b-a)^\alpha\|f^{(\alpha)}\|_\infty. \tag{2}$$

式(2)称为广义 Ostrowski 不等式[8],是 Ostrowski 不等式在分形集的推广.

设区间 $I\subseteq\mathbf{R}$,I° 是 I 的内部,$f:I^\circ\to\mathbf{R}^\alpha$,$f\in D_\alpha(I^\circ)$,$a,b\in I^\circ$,$a<b$,$f^{(\alpha)}\in C_\alpha[a,b]$,$w:[a,b]\to\mathbf{R}^\alpha$ 是非负的,$w(t)\in I_x^{(\alpha)}[a,b]$,则对任意 $x\in[a,b]$ 有

$$|[{}_aI_b^{(\alpha)}w(t)]f(x)-{}_aI_b^{(\alpha)}w(t)f(t)|\leqslant \frac{2^\alpha(b-a)^{2\alpha}}{\Gamma(1+2\alpha)}\left[\frac{1}{4^\alpha}+\left(\frac{x-\frac{a+b}{2}}{b-a}\right)^{2\alpha}\right]\|w\|_\infty\|f^{(\alpha)}\|_\infty. \tag{3}$$

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式(3)称为广义加权 Ostrowski 不等式[9]. 关于广义 Ostrowski 型不等式还可参见文献[10-14].

引理 1[7] (1) 设 $f(x)=g^{(\alpha)}(x)\in C_\alpha[a,b]$, 则 ${}_aI_b^{(\alpha)}f(x)=g(b)-g(a)$. 特别地,若 $f(x)$ 恒为常数 c,则有 ${}_aI_b^{(\alpha)}f(x)=\dfrac{c(b-a)^\alpha}{\Gamma(1+\alpha)}$.

(2) 局部分数阶定积分的分部积分法. 设 $f,g\in D_\alpha[a,b]$ 且 $f^{(\alpha)}(x),g^{(\alpha)}(x)\in C_\alpha[a,b]$, 则

$$ {}_aI_b^{(\alpha)}f(x)g^{(\alpha)}(x)=f(x)g(x)\mid_a^b-{}_aI_b^{(\alpha)}f^{(\alpha)}(x)g(x).$$

引理 2[15] 局部分数阶微分中值定理. 设函数 $F(x)$ 在 $[a,b]$ 上连续,在 (a,b) 上 α 阶局部分数阶可微,则对于任意 $x_0,x\in[a,b],x_0<x$, 存在 $\xi\in[x_0,x]$, 使得

$$F(x)=F(x_0)+F^{(\alpha)}(\xi)\frac{(x-x_0)^\alpha}{\Gamma(1+\alpha)}.$$

引理 3[16] 设 $f\in C_\alpha[a,b],0<\alpha\leqslant1$,积分上限函数定义为 $\Pi(x)={}_aI_x^{(\alpha)}f(t)$, 则 $\Pi\in D_\alpha[a,b]$, 且对于任意 $x\in[a,b]$ 有 $\Pi^{(\alpha)}(x)=f(x)$.

本文约定 $S_\alpha=\dfrac{f(b)-f(a)}{(b-a)^\alpha}$.

受参考文献[17]启发,本文用引入参数求最值的方法,建立分形集 $\mathbf{R}^\alpha$ 上加权 Ostrowski 型双边不等式. 为证明本文主要结果,还需要下面的引理.

引理 4[9] 设区间 $I\subseteq\mathbf{R},I^\circ$ 是 I 的内部, $f:I^\circ\to\mathbf{R}^\alpha,f\in D_\alpha(I^\circ),a,b\in I^\circ,a<b,f^{(\alpha)}\in C_\alpha[a,b],w:[a,b]\to\mathbf{R}^\alpha$ 是非负的, $w(t)\in I_x^{(\alpha)}[a,b]$, 则对任意 $x\in[a,b]$ 有

$$[{}_aI_b^{(\alpha)}w(t)]f(x)-{}_aI_b^{(\alpha)}w(t)f(t)=\frac{1}{\Gamma(1+\alpha)}\int_a^x\left[\frac{1}{\Gamma(1+\alpha)}\int_a^t w(u)(\mathrm{d}u)^\alpha\right]f^{(\alpha)}(t)(\mathrm{d}t)^\alpha+\frac{1}{\Gamma(1+\alpha)}\int_x^b\left[\frac{1}{\Gamma(1+\alpha)}\int_b^t w(u)(\mathrm{d}u)^\alpha\right]f^{(\alpha)}(t)(\mathrm{d}t)^\alpha.$$

1 主要结果

定理 1 设 $f\in D_\alpha[a,b],f^{(\alpha)}\in C_\alpha[a,b]$, 且存在常数 $\gamma,\Gamma\in\mathbf{R}^\alpha,\gamma<\Gamma$, 使得 $\gamma\leqslant f^{(\alpha)}\leqslant\Gamma$, 则有

$$B+\max\left\{0,-\frac{\Gamma-\gamma}{\Gamma^2(1+\alpha)}\int_a^{D(x)}(t-a-|\varepsilon_2|)^\alpha w(t)(\mathrm{d}t)^\alpha\right\}\leqslant B+\frac{\Gamma-\gamma}{\Gamma^2(1+\alpha)}\int_a^{a+|\varepsilon_2|}(a+|\varepsilon_2|-t)^\alpha w(t)(\mathrm{d}t)^\alpha\leqslant$$
$$[{}_aI_b^{(\alpha)}w(t)]f(x)-{}_aI_b^{(\alpha)}w(t)f(t)\leqslant A-\frac{\Gamma-\gamma}{\Gamma^2(1+\alpha)}\int_a^{a+|\varepsilon_1|}(a+|\varepsilon_1|-t)^\alpha w(t)(\mathrm{d}t)^\alpha\leqslant$$
$$A+\min\left\{0,\frac{\Gamma-\gamma}{\Gamma^2(1+\alpha)}\int_a^{C(x)}(t-a-|\varepsilon_1|)^\alpha w(t)(\mathrm{d}t)^\alpha\right\},\tag{4}$$

其中 $\varepsilon_1,\varepsilon_2\in[x-b,x-a]$ 满足

$$\varepsilon_1^\alpha=\frac{(x-a)^\alpha\Gamma+(b-x)^\alpha\gamma-(b-a)^\alpha\Gamma(1+\alpha)S_\alpha}{\Gamma-\gamma},\varepsilon_2^\alpha=\frac{(b-a)^\alpha\Gamma(1+\alpha)S_\alpha-(x-a)^\alpha\gamma-(b-x)^\alpha\Gamma}{\Gamma-\gamma},$$
$$A=\frac{\Gamma}{\Gamma^2(1+\alpha)}\int_a^x(x-t)w(t)(\mathrm{d}t)^\alpha-\frac{\gamma}{\Gamma^2(1+\alpha)}\int_x^b(t-x)w(t)(\mathrm{d}t)^\alpha,$$
$$B=\frac{\gamma}{\Gamma^2(1+\alpha)}\int_a^x(x-t)w(t)(\mathrm{d}t)^\alpha-\frac{\Gamma}{\Gamma^2(1+\alpha)}\int_x^b(t-x)w(t)(\mathrm{d}t)^\alpha,$$
$$C(x)=\begin{cases}x, & S_\alpha\leqslant\dfrac{(x-a)^\alpha\Gamma+(b-x)^\alpha\gamma}{(b-a)^\alpha\Gamma(1+\alpha)},\\ a+b-x, & S_\alpha>\dfrac{(x-a)^\alpha\Gamma+(b-x)^\alpha\gamma}{(b-a)^\alpha\Gamma(1+\alpha)},\end{cases}$$
$$D(x)=\begin{cases}x, & S_\alpha\geqslant\dfrac{(x-a)^\alpha\gamma+(b-x)^\alpha\Gamma}{(b-a)^\alpha\Gamma(1+\alpha)},\\ a+b-x, & S_\alpha<\dfrac{(x-a)^\alpha\gamma+(b-x)^\alpha\Gamma}{(b-a)^\alpha\Gamma(1+\alpha)}.\end{cases}$$

证明 当 $S_\alpha\leqslant\dfrac{(x-a)^\alpha\Gamma+(b-x)^\alpha\gamma}{(b-a)^\alpha\Gamma(1+\alpha)}$ 时,对任意 $\varepsilon\in[0,x-a]$, 利用引理 4 有

$$f(x){}_aI_b^{(\alpha)}w(t)-{}_aI_b^{(\alpha)}w(t)f(t)-S_\alpha(b-a)^\alpha{}_aI_{a+\varepsilon}^{(\alpha)}w(t)=$$

$$\frac{1}{\Gamma(1+\alpha)}\int_a^x\left[\frac{1}{\Gamma(1+\alpha)}\int_a^t w(u)(\mathrm{d}u)^\alpha\right]f^{(\alpha)}(t)(\mathrm{d}t)^\alpha+\frac{1}{\Gamma(1+\alpha)}\int_x^b\left[\frac{1}{\Gamma(1+\alpha)}\int_b^t w(u)(\mathrm{d}u)^\alpha\right]f^{(\alpha)}(t)(\mathrm{d}t)^\alpha-$$
$$\frac{1}{\Gamma(1+\alpha)}\int_a^x f^{(\alpha)}(t)(\mathrm{d}t)^\alpha\frac{1}{\Gamma(1+\alpha)}\int_a^{a+\varepsilon}w(u)(\mathrm{d}u)^\alpha-\frac{1}{\Gamma(1+\alpha)}\int_x^b f^{(\alpha)}(t)(\mathrm{d}t)^\alpha\frac{1}{\Gamma(1+\alpha)}\int_a^{a+\varepsilon}w(u)(\mathrm{d}u)^\alpha=$$
$$\frac{1}{\Gamma^2(1+\alpha)}\int_a^x\left[\int_{a+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\right]f^{(\alpha)}(t)(\mathrm{d}t)^\alpha+\frac{1}{\Gamma^2(1+\alpha)}\int_x^b\left[\int_b^t w(u)(\mathrm{d}u)^\alpha\right]f^{(\alpha)}(t)(\mathrm{d}t)^\alpha-$$
$$\frac{1}{\Gamma(1+\alpha)}\int_x^b f^{(\alpha)}(t)(\mathrm{d}t)^\alpha[{}_aI_{a+\varepsilon}^{(\alpha)}w(t)]\leqslant$$
$$\frac{\gamma}{\Gamma^2(1+\alpha)}\int_a^{a+\varepsilon}\left[\int_{a+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\right](\mathrm{d}t)^\alpha+\frac{\Gamma}{\Gamma^2(1+\alpha)}\int_{a+\varepsilon}^x\left[\int_{a+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\right](\mathrm{d}t)^\alpha+$$
$$\frac{\gamma}{\Gamma^2(1+\alpha)}\int_x^b\left[\int_b^t w(u)(\mathrm{d}u)^\alpha\right](\mathrm{d}t)^\alpha-\frac{\gamma(b-x)^\alpha}{\Gamma(1+\alpha)}[{}_aI_{a+\varepsilon}^{(\alpha)}w(t)]. \tag{5}$$

利用引理 1(局部分数阶分部积分法)得

$$\frac{1}{\Gamma(1+\alpha)}\int_a^{a+\varepsilon}\left[\int_{a+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\right](\mathrm{d}t)^\alpha=\frac{1}{\Gamma(1+\alpha)}\int_a^{a+\varepsilon}\left[\frac{1}{\Gamma(1+\alpha)}\int_{a+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\right]((t-a)^\alpha)^{(\alpha)}(\mathrm{d}t)^\alpha=$$
$$\frac{(t-a)^\alpha}{\Gamma(1+\alpha)}\int_{a+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\Big|_a^{a+\varepsilon}-\frac{1}{\Gamma(1+\alpha)}\int_a^{a+\varepsilon}(t-a)^\alpha w(t)(\mathrm{d}t)^\alpha=-\frac{1}{\Gamma(1+\alpha)}\int_a^{a+\varepsilon}(t-a)^\alpha w(t)(\mathrm{d}t)^\alpha, \tag{6}$$

$$\frac{1}{\Gamma(1+\alpha)}\int_{a+\varepsilon}^x\left[\int_{a+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\right](\mathrm{d}t)^\alpha=\frac{1}{\Gamma(1+\alpha)}\int_{a+\varepsilon}^x\left[\frac{1}{\Gamma(1+\alpha)}\int_{a+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\right]((t-x)^\alpha)^{(\alpha)}(\mathrm{d}t)^\alpha=$$
$$\frac{(t-x)^\alpha}{\Gamma(1+\alpha)}\int_{a+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\Big|_{a+\varepsilon}^x-\frac{1}{\Gamma(1+\alpha)}\int_{a+\varepsilon}^x(t-x)^\alpha w(t)(\mathrm{d}t)^\alpha=\frac{1}{\Gamma(1+\alpha)}\int_{a+\varepsilon}^x(x-t)^\alpha w(t)(\mathrm{d}t)^\alpha, \tag{7}$$

$$\frac{1}{\Gamma(1+\alpha)}\int_x^b\left[\int_b^t w(u)(\mathrm{d}u)^\alpha\right](\mathrm{d}t)^\alpha=\frac{1}{\Gamma(1+\alpha)}\int_x^b\left[\frac{1}{\Gamma(1+\alpha)}\int_b^t w(u)(\mathrm{d}u)^\alpha\right]((t-x)^\alpha)^{(\alpha)}(\mathrm{d}t)^\alpha=$$
$$\frac{(t-x)^\alpha}{\Gamma(1+\alpha)}\int_b^t w(u)(\mathrm{d}u)^\alpha\Big|_x^b-\frac{1}{\Gamma(1+\alpha)}\int_x^b(t-x)^\alpha w(t)(\mathrm{d}t)^\alpha=-\frac{1}{\Gamma(1+\alpha)}\int_x^b(t-x)^\alpha w(t)(\mathrm{d}t)^\alpha, \tag{8}$$

综合式(5)~(8)得

$$f(x)\,{}_aI_b^{(\alpha)}w(t)-{}_aI_b^{(\alpha)}w(t)f(t)\leqslant\frac{1}{\Gamma(1+\alpha)}\varphi_1(\varepsilon),$$

其中

$$\varphi_1(\varepsilon)=-\frac{\gamma}{\Gamma(1+\alpha)}\int_a^{a+\varepsilon}(t-a)^\alpha w(t)(\mathrm{d}t)^\alpha+\frac{\Gamma}{\Gamma(1+\alpha)}\int_{a+\varepsilon}^x(x-t)^\alpha w(t)(\mathrm{d}t)^\alpha-$$
$$\frac{\gamma}{\Gamma(1+\alpha)}\int_x^b(t-x)^\alpha w(t)(\mathrm{d}t)^\alpha+[(b-a)^\alpha\Gamma(1+\alpha)S_\alpha-(b-x)^\alpha\gamma]\,{}_aI_{a+\varepsilon}^{(\alpha)}w(t),$$

于是有

$$f(x)\,{}_aI_b^{(\alpha)}w(t)-{}_aI_b^{(\alpha)}w(t)f(t)\leqslant\frac{1}{\Gamma(1+\alpha)}\min_{\varepsilon\in[0,x-a]}\varphi_1(\varepsilon)\leqslant\frac{1}{\Gamma(1+\alpha)}\min\{\varphi_1(0),\varphi_1(x-a)\}. \tag{9}$$

$$\varphi_1(0)=\frac{\Gamma}{\Gamma(1+\alpha)}\int_a^x(x-t)^\alpha w(t)(\mathrm{d}t)^\alpha-\frac{\gamma}{\Gamma(1+\alpha)}\int_x^b(t-x)^\alpha w(t)(\mathrm{d}t)^\alpha, \tag{10}$$

$$\varphi_1(x-a)=\frac{1}{\Gamma(1+\alpha)}[\Gamma\int_a^x(x-t)^\alpha w(t)(\mathrm{d}t)^\alpha-\gamma\int_x^b(t-x)^\alpha w(t)(\mathrm{d}t)^\alpha+(\Gamma-\gamma)\int_a^x(t-a-\varepsilon_1)^\alpha w(t)(\mathrm{d}t)^\alpha]. \tag{11}$$

利用引理 3 得 $\varphi_1^{(\alpha)}(\varepsilon)=w(a+\varepsilon)(\Gamma-\gamma)(\varepsilon-\varepsilon_1)^\alpha$,对任意 $\varepsilon\in[0,x-a]$,由引理 2,在 ε 与 ε_1 之间存在 ξ 使得 $\varphi_1(\varepsilon)-\varphi_1(\varepsilon_1)=\frac{\varphi_1^{(\alpha)}(\xi)}{\Gamma(1+\alpha)}(\varepsilon-\varepsilon_1)^\alpha=\frac{w(a+\xi)}{\Gamma(1+\alpha)}(\Gamma-\gamma)(\xi-\varepsilon_1)^\alpha(\varepsilon-\varepsilon_1)^\alpha\geqslant0$,故 $\varphi_1(\varepsilon)$

在 ε_1 处取得最小值,且

$$\varphi_1(\varepsilon_1) = -\frac{\gamma}{\Gamma(1+\alpha)}\int_a^{a+\varepsilon_1}(t-a)^\alpha w(t)(\mathrm{d}t)^\alpha + \frac{\Gamma}{\Gamma(1+\alpha)}\int_{a+\varepsilon_1}^x (x-t)^\alpha w(t)(\mathrm{d}t)^\alpha -$$

$$\frac{\gamma}{\Gamma(1+\alpha)}\int_x^b (t-x)^\alpha w(t)(\mathrm{d}t)^\alpha + [(b-a)^\alpha\Gamma(1+\alpha)S_\alpha - (b-x)^\alpha\gamma]\,{}_aI_{a+\varepsilon_1}^{(\alpha)}w(t) =$$

$$-\frac{\gamma}{\Gamma(1+\alpha)}\int_a^{a+\varepsilon_1}(t-a)^\alpha w(t)(\mathrm{d}t)^\alpha + \frac{\Gamma}{\Gamma(1+\alpha)}\int_a^x (x-t)^\alpha w(t)(\mathrm{d}t)^\alpha - \frac{\Gamma}{\Gamma(1+\alpha)}\int_a^{a+\varepsilon_1}(x-t)^\alpha w(t)(\mathrm{d}t)^\alpha -$$

$$\frac{\gamma}{\Gamma(1+\alpha)}\int_x^b (t-x)^\alpha w(t)(\mathrm{d}t)^\alpha + [(x-a)^\alpha\Gamma - (\Gamma-\gamma)\varepsilon_1^\alpha]\,{}_aI_{a+\varepsilon_1}^{(\alpha)}w(t) =$$

$$\frac{\Gamma}{\Gamma(1+\alpha)}\int_a^x (x-t)^\alpha w(t)(\mathrm{d}t)^\alpha - \frac{\gamma}{\Gamma(1+\alpha)}\int_x^b (t-x)^\alpha w(t)(\mathrm{d}t)^\alpha - \frac{\Gamma-\gamma}{\Gamma(1+\alpha)}\int_a^{a+\varepsilon_1}(a+\varepsilon_1-t)^\alpha w(t)(\mathrm{d}t)^\alpha, \tag{12}$$

综合式(9)~(12)得

$$[\,{}_aI_b^{(\alpha)}w(t)]f(x) - {}_aI_b^{(\alpha)}w(t)f(t) \leqslant A - \frac{\Gamma-\gamma}{\Gamma^2(1+\alpha)}\int_a^{a+\varepsilon_1}(a+\varepsilon_1-t)^\alpha w(t)(\mathrm{d}t)^\alpha \leqslant$$

$$A + \min\left\{0, \frac{\Gamma-\gamma}{\Gamma^2(1+\alpha)}\int_a^x (t-a-\varepsilon_1)^\alpha w(t)(\mathrm{d}t)^\alpha\right\}. \tag{13}$$

当 $S_\alpha > \dfrac{(x-a)^\alpha\Gamma + (b-x)^\alpha\gamma}{(b-a)^\alpha\Gamma(1+\alpha)}$ 时,对任意 $\varepsilon \in [x-b,0]$,利用引理 4 得

$$f(x)\,{}_aI_b^{(\alpha)}w(t) - {}_aI_b^{(\alpha)}w(t)f(t) + S_\alpha(b-a)^\alpha\,{}_{b+\varepsilon}I_b^{(\alpha)}w(t) =$$

$$\frac{1}{\Gamma(1+\alpha)}\int_a^x\left[\frac{1}{\Gamma(1+\alpha)}\int_a^t w(u)(\mathrm{d}u)^\alpha\right]f^{(\alpha)}(t)(\mathrm{d}t)^\alpha + \frac{1}{\Gamma(1+\alpha)}\int_x^b\left[\frac{1}{\Gamma(1+\alpha)}\int_b^t w(u)(\mathrm{d}u)^\alpha\right]f^{(\alpha)}(t)(\mathrm{d}t)^\alpha +$$

$$\frac{1}{\Gamma(1+\alpha)}\int_a^x f^{(\alpha)}(t)(\mathrm{d}t)^\alpha\frac{1}{\Gamma(1+\alpha)}\int_{b+\varepsilon}^b w(u)(\mathrm{d}u)^\alpha + \frac{1}{\Gamma(1+\alpha)}\int_x^b f^{(\alpha)}(t)(\mathrm{d}t)^\alpha\frac{1}{\Gamma(1+\alpha)}\int_{b+\varepsilon}^b w(u)(\mathrm{d}u)^\alpha =$$

$$\frac{1}{\Gamma^2(1+\alpha)}\int_a^x\left[\int_a^t w(u)(\mathrm{d}u)^\alpha\right]f^{(\alpha)}(t)(\mathrm{d}t)^\alpha + \frac{1}{\Gamma(1+\alpha)}\int_a^x f^{(\alpha)}(t)(\mathrm{d}t)^\alpha[\,I_{b+\varepsilon}^b w(t)] +$$

$$\frac{1}{\Gamma^2(1+\alpha)}\int_x^b\left[\int_{b+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\right]f^{(\alpha)}(t)(\mathrm{d}t)^\alpha \leqslant \frac{\Gamma}{\Gamma^2(1+\alpha)}\int_a^x\left[\int_a^t w(u)(\mathrm{d}u)^\alpha\right](\mathrm{d}t)^\alpha + \frac{(x-a)^\alpha\Gamma}{\Gamma(1+\alpha)}[\,I_{b+\varepsilon}^b w(t)] +$$

$$\frac{\gamma}{\Gamma^2(1+\alpha)}\int_x^{b+\varepsilon}\left[\int_{b+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\right](\mathrm{d}t)^\alpha + \frac{\Gamma}{\Gamma^2(1+\alpha)}\int_{b+\varepsilon}^b\left[\int_{b+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\right](\mathrm{d}t)^\alpha. \tag{14}$$

利用引理 1(局部分数阶分部积分法)得

$$\frac{1}{\Gamma(1+\alpha)}\int_a^x\left[\int_a^t w(u)(\mathrm{d}u)^\alpha\right](\mathrm{d}t)^\alpha = \frac{1}{\Gamma(1+\alpha)}\int_a^x\left[\frac{1}{\Gamma(1+\alpha)}\int_a^t w(u)(\mathrm{d}u)^\alpha\right]((t-x)^\alpha)^{(\alpha)}(\mathrm{d}t)^\alpha =$$

$$\frac{(t-x)^\alpha}{\Gamma(1+\alpha)}\int_a^t w(u)(\mathrm{d}u)^\alpha\Big|_a^x - \frac{1}{\Gamma(1+\alpha)}\int_a^x (t-x)^\alpha w(t)(\mathrm{d}t)^\alpha = \frac{1}{\Gamma(1+\alpha)}\int_a^x (x-t)^\alpha w(t)(\mathrm{d}t)^\alpha, \tag{15}$$

$$\frac{1}{\Gamma(1+\alpha)}\int_x^{b+\varepsilon}\left[\int_{b+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\right](\mathrm{d}t)^\alpha = \frac{1}{\Gamma(1+\alpha)}\int_x^{b+\varepsilon}\left[\frac{1}{\Gamma(1+\alpha)}\int_{b+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\right]((t-x)^\alpha)^{(\alpha)}(\mathrm{d}t)^\alpha =$$

$$\frac{(t-x)^\alpha}{\Gamma(1+\alpha)}\int_{b+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\Big|_x^{b+\varepsilon} - \frac{1}{\Gamma(1+\alpha)}\int_x^{b+\varepsilon}(t-x)^\alpha w(t)(\mathrm{d}t)^\alpha = -\frac{1}{\Gamma(1+\alpha)}\int_x^{b+\varepsilon}(t-x)^\alpha w(t)(\mathrm{d}t)^\alpha, \tag{16}$$

$$\frac{1}{\Gamma(1+\alpha)}\int_{b+\varepsilon}^b\left[\int_{b+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\right](\mathrm{d}t)^\alpha = \frac{1}{\Gamma(1+\alpha)}\int_{b+\varepsilon}^b\left[\frac{1}{\Gamma(1+\alpha)}\int_{b+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\right]((t-b)^\alpha)^{(\alpha)}(\mathrm{d}t)^\alpha =$$

$$\frac{(t-b)^\alpha}{\Gamma(1+\alpha)}\int_{b+\varepsilon}^t w(u)(\mathrm{d}u)^\alpha\Big|_{b+\varepsilon}^b - \frac{1}{\Gamma(1+\alpha)}\int_{b+\varepsilon}^b (t-b)^\alpha w(t)(\mathrm{d}t)^\alpha = \frac{1}{\Gamma(1+\alpha)}\int_{b+\varepsilon}^b (b-t)^\alpha w(t)(\mathrm{d}t)^\alpha, \tag{17}$$

综合式(14)~(17)得

$$f(x)\,{}_aI_b^{(\alpha)}w(t) - {}_aI_b^{(\alpha)}w(t)f(t) \leqslant \frac{1}{\Gamma(1+\alpha)}\varphi_2(\varepsilon),$$

其中

$$\varphi_2(\varepsilon) = \frac{\Gamma}{\Gamma(1+\alpha)}\int_a^x (x-t)^\alpha w(t)(\mathrm{d}t)^\alpha - \frac{\gamma}{\Gamma(1+\alpha)}\int_x^{b+\varepsilon}(t-x)^\alpha w(t)(\mathrm{d}t)^\alpha +$$

$$\frac{\Gamma}{\Gamma(1+\alpha)}\int_{b+\varepsilon}^b (b-t)^\alpha w(t)(\mathrm{d}t)^\alpha + [(x-a)^\alpha\Gamma - (b-a)^\alpha\Gamma(1+\alpha)S_\alpha]\,{}_{b+\varepsilon}I_b^{(\alpha)}w(t),$$

于是有

$$f(x)\,{}_aI_b^{(\alpha)}w(t) - {}_aI_b^{(\alpha)}w(t)f(t) \leqslant \frac{1}{\Gamma(1+\alpha)}\min_{\varepsilon\in[x-b,0]}\varphi_2(\varepsilon) \leqslant \frac{1}{\Gamma(1+\alpha)}\min\{\varphi_2(0),\varphi_2(x-b)\}, \tag{18}$$

$$\varphi_2(0) = \frac{\Gamma}{\Gamma(1+\alpha)}\int_a^x (x-t)^\alpha w(t)(\mathrm{d}t)^\alpha - \frac{\gamma}{\Gamma(1+\alpha)}\int_x^b (t-x)^\alpha w(t)(\mathrm{d}t)^\alpha, \tag{19}$$

$$\varphi_2(x-b) = \frac{1}{\Gamma(1+\alpha)}\left[\Gamma\int_a^x (x-t)^\alpha w(t)(\mathrm{d}t)^\alpha - \gamma\int_x^b (t-x)^\alpha w(t)(\mathrm{d}t)^\alpha + (\Gamma-\gamma)\int_a^{a+b-x}(t-a+\varepsilon_1)^\alpha w(t)(\mathrm{d}t)^\alpha\right]. \tag{20}$$

利用引理 3 得 $\varphi_2^{(\alpha)}(\varepsilon) = w(b+\varepsilon)(\Gamma-\gamma)(\varepsilon-\varepsilon_1)^\alpha$, 对任意 $\varepsilon\in[x-b,0]$, 由引理 2,在 ε 与 ε_1 之间存在 ξ, 使得 $\varphi_2(\varepsilon)-\varphi_2(\varepsilon_1) = \frac{\varphi_2^{(\alpha)}(\xi)}{\Gamma(1+\alpha)}(\varepsilon-\varepsilon_1)^\alpha = \frac{w(b+\xi)}{\Gamma(1+\alpha)}(\Gamma-\gamma)(\xi-\varepsilon_1)^\alpha(\varepsilon-\varepsilon_1)^\alpha \geqslant 0$,故 $\varphi_2(\varepsilon)$ 在 ε_1 处取得最小值,且

$$\varphi_2(\varepsilon_1) = \frac{\Gamma}{\Gamma(1+\alpha)}\int_a^x (x-t)^\alpha w(t)(\mathrm{d}t)^\alpha - \frac{\gamma}{\Gamma(1+\alpha)}\int_x^b (t-x)^\alpha w(t)(\mathrm{d}t)^\alpha + \frac{\gamma}{\Gamma(1+\alpha)}\int_{b+\varepsilon}^b (t-x)^\alpha w(t)(\mathrm{d}t)^\alpha +$$

$$\frac{\Gamma}{\Gamma(1+\alpha)}\int_{b+\varepsilon}^b (b-t)^\alpha w(t)(\mathrm{d}t)^\alpha + [(\Gamma-\gamma)\varepsilon_1^\alpha - (b-x)^\alpha\gamma]\,{}_{b+\varepsilon}I_b^{(\alpha)}w(t) =$$

$$\frac{\Gamma}{\Gamma(1+\alpha)}\int_a^x (x-t)^\alpha w(t)(\mathrm{d}t)^\alpha - \frac{\gamma}{\Gamma(1+\alpha)}\int_x^b (t-x)^\alpha w(t)(\mathrm{d}t)^\alpha - \frac{\Gamma-\gamma}{\Gamma(1+\alpha)}\int_{b+\varepsilon_1}^b (t-b-\varepsilon_1)^\alpha w(t)(\mathrm{d}t)^\alpha =$$

$$\frac{\Gamma}{\Gamma(1+\alpha)}\int_a^x (x-t)^\alpha w(t)(\mathrm{d}t)^\alpha - \frac{\gamma}{\Gamma(1+\alpha)}\int_x^b (t-x)^\alpha w(t)(\mathrm{d}t)^\alpha - \frac{\Gamma-\gamma}{\Gamma(1+\alpha)}\int_a^{a-\varepsilon_1}(a-\varepsilon_1-t)^\alpha w(t)(\mathrm{d}t)^\alpha, \tag{21}$$

综合式(18)~(21)有

$$[{}_aI_b^{(\alpha)}w(t)]f(x) - {}_aI_b^{(\alpha)}w(t)f(t) \leqslant A - \frac{\Gamma-\gamma}{\Gamma^2(1+\alpha)}\int_a^{a-\varepsilon_1}(a-\varepsilon_1-t)^\alpha w(t)(\mathrm{d}t)^\alpha \leqslant$$

$$A + \min\left\{0, \frac{\Gamma-\gamma}{\Gamma^2(1+\alpha)}\int_a^{a+b-x}(t-a+\varepsilon_1)^\alpha w(t)(\mathrm{d}t)^\alpha\right\}, \tag{22}$$

综合式(13)和(22),则式(4)的右边 2 个不等式得证.

当 $\gamma \leqslant f^{(\alpha)} \leqslant \Gamma$ 时,有 $-\Gamma \leqslant (-f)^{(\alpha)} \leqslant -\gamma$. 对函数 $-f$ 应用已证结论,则式(4)的左边 2 个不等式得证.

推论 1　设 $f\in D_\alpha[a,b]$, $f^{(\alpha)}\in C_\alpha[a,b]$, 且存在常数 $M\in\mathbf{R}^\alpha$, 使得 $|f^{(\alpha)}|\leqslant M$, $w:[a,b]\to\mathbf{R}^\alpha$ 是非负的, $w(t)\in I_x^{(\alpha)}[a,b]$, 则有

$$-\frac{M}{\Gamma^2(1+\alpha)}\int_a^b |t-x|^\alpha w(t)(\mathrm{d}t)^\alpha + \max\left\{0, -\frac{2^\alpha M}{\Gamma^2(1+\alpha)}\int_a^{D(x)}(t-a-|\varepsilon_2|)^\alpha w(t)(\mathrm{d}t)^\alpha\right\} \leqslant$$

$$-\frac{M}{\Gamma^2(1+\alpha)}\int_a^b |t-x|^\alpha w(t)(\mathrm{d}t)^\alpha + \frac{2^\alpha M}{\Gamma^2(1+\alpha)}\int_a^{a+|\varepsilon_2|}(a+|\varepsilon_2|-t)^\alpha w(t)(\mathrm{d}t)^\alpha \leqslant$$

$$[{}_aI_b^{(\alpha)}w(t)]f(x) - {}_aI_b^{(\alpha)}w(t)f(t) \leqslant \frac{M}{\Gamma^2(1+\alpha)}\int_a^b |t-x|^\alpha w(t)(\mathrm{d}t)^\alpha - \frac{2^\alpha M}{\Gamma^2(1+\alpha)}\int_a^{a+|\varepsilon_1|}(a+|\varepsilon_1|-t)^\alpha w(t)(\mathrm{d}t)^\alpha \leqslant$$

$$\frac{M}{\Gamma^2(1+\alpha)}\int_a^b |t-x|^\alpha w(t)(\mathrm{d}t)^\alpha + \min\left\{0, \frac{2^\alpha M}{\Gamma^2(1+\alpha)}\int_a^{C(x)}(t-a-|\varepsilon_1|)^\alpha w(t)(\mathrm{d}t)^\alpha\right\},$$

其中 $\varepsilon_1,\varepsilon_2\in[x-b,x-a]$ 满足

$$\varepsilon_1^{\alpha}=\frac{(2x-a-b)^{\alpha}M-(b-a)^{\alpha}\Gamma(1+\alpha)S_{\alpha}}{2^{\alpha}M},\quad \varepsilon_2^{\alpha}=\frac{(2x-a-b)^{\alpha}M+(b-a)^{\alpha}\Gamma(1+\alpha)S_{\alpha}}{2^{\alpha}M},$$

$$C(x)=\begin{cases}x, & S_{\alpha}\leqslant\dfrac{(2x-a-b)^{\alpha}M}{(b-a)^{\alpha}\Gamma(1+\alpha)},\\ a+b-x, & S_{\alpha}>\dfrac{(2x-a-b)^{\alpha}M}{(b-a)^{\alpha}\Gamma(1+\alpha)},\end{cases}\quad D(x)=\begin{cases}x, & S_{\alpha}\geqslant\dfrac{(2x-a-b)^{\alpha}M}{(b-a)^{\alpha}\Gamma(1+\alpha)},\\ a+b-x, & S_{\alpha}<\dfrac{(2x-a-b)^{\alpha}M}{(b-a)^{\alpha}\Gamma(1+\alpha)}.\end{cases}$$

推论 2 设 $f\in D_{\alpha}[a,b]$, $f^{(\alpha)}\in C_{\alpha}[a,b]$, 且存在常数 $M\in\mathbf{R}^{\alpha}$, 使得 $|f^{(\alpha)}|\leqslant M$, 则有

$$\left|f(x)-\frac{\Gamma^2(1+\alpha)}{\Gamma(1+2\alpha)}(2x-a-b)^{\alpha}S_{\alpha}-\frac{\Gamma(1+\alpha)}{(b-a)^{\alpha}}{}_aI_b^{(\alpha)}f(t)\right|\leqslant\frac{\Gamma(1+\alpha)(b-a)^{\alpha}}{2^{\alpha}\Gamma(1+2\alpha)}\left(M-\frac{\Gamma^2(1+\alpha)S_{\alpha}^2}{M}\right).$$

证明 在推论 1 中取 $w(t)\equiv 1^{\alpha}$, 经计算则推论 2 得证.

推论 3 设 $f\in D_{\alpha}[a,b]$, $f^{(\alpha)}\in C_{\alpha}[a,b]$, 且存在常数 $\gamma,\Gamma\in\mathbf{R}^{\alpha}$, $\gamma<\Gamma$, 使得 $\gamma\leqslant f^{(\alpha)}\leqslant\Gamma$, 则有

$$\left|f(x)+(\Gamma+\gamma)\left(\frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}-\frac{1}{2^{\alpha}\Gamma(1+\alpha)}\right)x^{\alpha}-\frac{\Gamma^2(1+\alpha)}{\Gamma(1+2\alpha)}(2x-a-b)^{\alpha}S_{\alpha}-\frac{\Gamma(1+\alpha)}{(b-a)^{\alpha}}{}_aI_b^{(\alpha)}f(t)\right|\leqslant$$

$$\frac{\Gamma(1+\alpha)(b-a)^{\alpha}}{4^{\alpha}\Gamma(1+2\alpha)}\left(\Gamma-\gamma-\frac{4^{\alpha}\Gamma^2(1+\alpha)S_{\alpha}^2}{\Gamma-\gamma}\right).$$

证明 因为 $\gamma\leqslant f^{(\alpha)}\leqslant\Gamma$, 所以有 $\left|f^{(\alpha)}-\dfrac{\Gamma+\gamma}{2^{\alpha}}\right|\leqslant\dfrac{\Gamma-\gamma}{2^{\alpha}}$. 对函数 $f(t)-\dfrac{\Gamma+\gamma}{2^{\alpha}\Gamma(1+\alpha)}t^{\alpha}$ 使用推论 2, 则推论 3 得证.

推论 4 设 $f:[a,b]\to\mathbf{R}$ 在 $[a,b]$ 上连续, 在 (a,b) 上可微, 且存在常数 $\gamma,\Gamma\in\mathbf{R}$, $\gamma<\Gamma$, 使得 $\gamma\leqslant f'\leqslant\Gamma$, 则有

$$\left|f(x)-\left(\frac{x-(a+b)/2}{b-a}\right)[f(b)-f(a)]-\frac{1}{b-a}\int_a^b f(t)\,\mathrm{d}t\right|\leqslant\frac{1}{8}(b-a)\left(\Gamma-\gamma-\frac{4(f(b)-f(a))^2}{(\Gamma-\gamma)(b-a)^2}\right).\tag{23}$$

证明 在推论 3 中取 $\alpha=1$, 则推论 4 得证.

注 1 式(23)给出文献[2]中的 Ostrowski-Grüss 型不等式(8.10)的加强.

推论 5 设 $f\in D_{\alpha}[a,b]$, $f^{(\alpha)}\in C_{\alpha}[a,b]$, 且存在常数 $M\in\mathbf{R}^{\alpha}$, 使得 $|f^{(\alpha)}|\leqslant M$, $w:[a,b]\to\mathbf{R}^{\alpha}$ 是非负的, $w(t)\in I_x^{(\alpha)}[a,b]$, 则有

$$\left|f\left(\frac{a+b}{2}\right){}_aI_b^{(\alpha)}w(t)-{}_aI_b^{(\alpha)}w(t)f(t)\right|\leqslant$$

$$\frac{M}{\Gamma^2(1+\alpha)}\int_a^b\left|t-\frac{a+b}{2}\right|^{\alpha}w(t)(\mathrm{d}t)^{\alpha}-\frac{2^{\alpha}M}{\Gamma^2(1+\alpha)}\int_a^{a+k}(a+k-t)^{\alpha}w(t)(\mathrm{d}t)^{\alpha}\leqslant$$

$$\frac{M}{\Gamma^2(1+\alpha)}\int_a^b\left|t-\frac{a+b}{2}\right|^{\alpha}w(t)(\mathrm{d}t)^{\alpha}+\min\left\{0,\frac{2^{\alpha}M}{\Gamma^2(1+\alpha)}\int_a^{\frac{a+b}{2}}(t-a-k)^{\alpha}w(t)(\mathrm{d}t)^{\alpha}\right\}.$$

特别地, 当 $w(t)\equiv 1^{\alpha}$ 时, 有

$$\left|f\left(\frac{a+b}{2}\right)-\frac{\Gamma(1+\alpha)}{(b-a)^{\alpha}}{}_aI_b^{(\alpha)}f(t)\right|\leqslant\frac{(b-a)^{\alpha}\Gamma(1+\alpha)}{2^{\alpha}\Gamma(1+2\alpha)}\left(M-\Gamma^2(1+\alpha)\frac{S_{\alpha}^2}{M}\right)\leqslant$$

$$\frac{(b-a)^{\alpha}\Gamma(1+\alpha)M}{\Gamma(1+2\alpha)}\min\left\{\frac{1}{2^{\alpha}},1^{\alpha}-\frac{\Gamma(1+\alpha)}{M}|S_{\alpha}|\right\}.$$

证明 在推论 1 中取 $x=\dfrac{a+b}{2}$ 经简单计算即可得证.

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Two-sided Weighted Ostrowski Type Inequalities on Fractal Sets

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Abstract: Based on the theory of local fractional calculus on fractal sets, two-sided weighted Ostrowski type inequalities for functions whose first order local fractional derivatives are bounded on fractal sets are obtained by establishing the identity and using the method of introducing parameter to find the maximum.

Keywords: Ostrowski type inequality; local fractional derivative; local fractional integral; fractal set; weighted function